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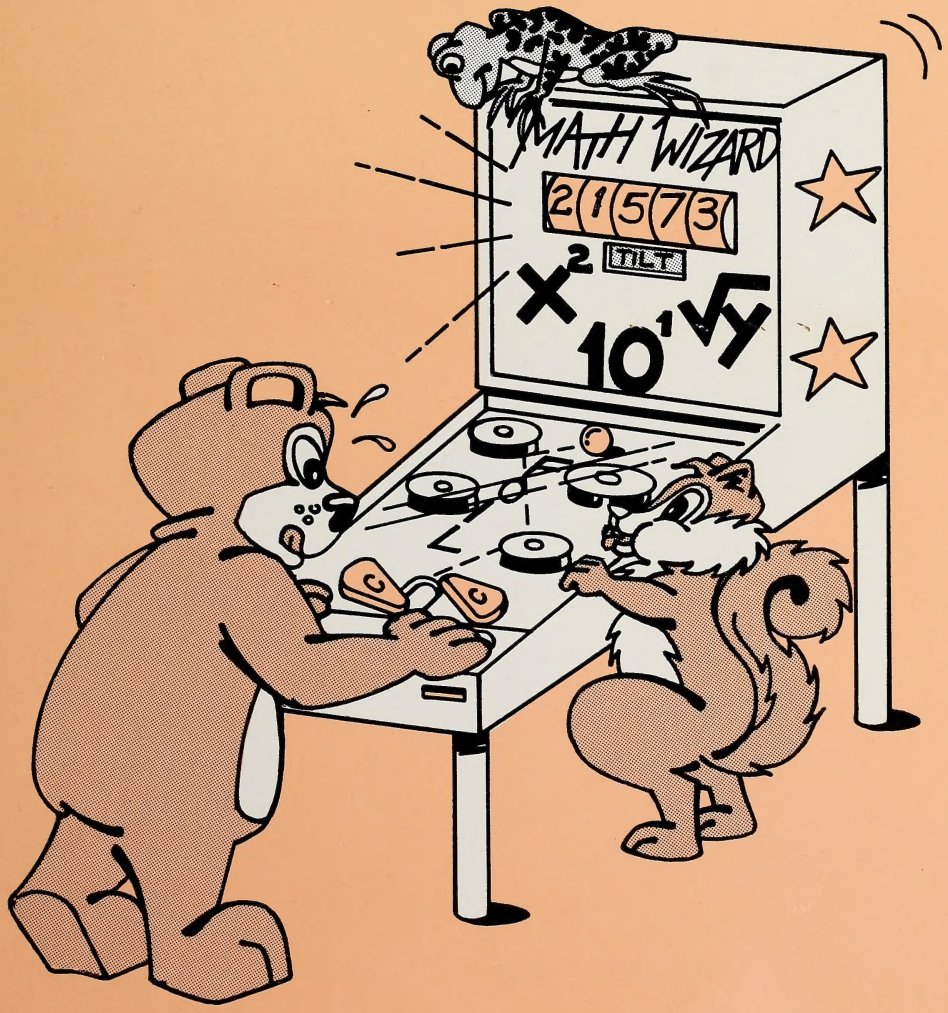
MATHEMATICS 9

BASED ON HOLT MATHEMATICS 3



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MATHEMATICS 9

Holt

LESSONS 1-30

Alberta
EDUCATION

Mathematics 9 (Holt)
Student Module
Lessons 1-30
Alberta Distance Learning Centre
ISBN No. 0-7741-0225-X

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Lesson 1 INTEGERS

- Integers are added, subtracted, multiplied, and divided. The rules for order of operations are discussed.

Lesson 2 POWERS

- Positive and negative powers are evaluated. The following laws are applied: $x^a \cdot x^b = x^{a+b}$; $x^a \div x^b = x^{a-b}$; $(x^a)^b = x^{a*b}$; $x^{-a} = \frac{1}{x^a}$.

Lesson 3 FRACTIONS AND RATIONAL NUMBERS

- Calculations are made for Greatest Common Factor and Lowest Common Multiple. Improper fractions are changed to mixed numbers and vice versa. Fractions, mixed numbers and rational numbers are added and subtracted.

Lesson 4 OPERATIONS WITH FRACTIONS, DECIMALS AND RATIONAL NUMBERS

- Fractions, mixed numbers and rational numbers are multiplied and divided. The writing of decimal numbers is discussed as well as rounding off of decimals. Finally addition, subtraction, multiplication, and division of decimals is studied.

Lesson 5 SOLVING EQUATIONS

- The following types of equations are solved: $x + a = b$, $ax = b$, $ax + c = b$, $ax + bx = c$, and $ax + b = cx + d$. English expressions are translated into algebraic expressions. Algebraic expressions are used in problem solving.

Lesson 6 FORMS OF WRITING RATIONAL NUMBERS

- Multiplication and division are done with powers of ten. Numbers are written in scientific notation. Fractions are changed to decimals and vice versa.

Lesson 7 RATIO AND PROPORTION

- A review is made of proportions using whole and rational numbers, fractions, percents and decimals. Three different types of percent problems are solved.

Lesson 8 SOLVING EQUATIONS

- Equations of the type $\frac{x}{a} = b$ and $\frac{x}{a} = \frac{b}{c}$ are solved. Other equations are solved using the distributive property and LCM.

Lesson 9 PROBLEM SOLVING

- Problem solving strategies are introduced. Number problems and measurement problems are solved.

Lesson 10 REVIEW

- This lesson reviews the concepts covered in Lessons 1 to 9.

Lesson 11 NUMBER SYSTEMS AND THEIR GRAPHS

- Graphing of whole numbers, integers, rational numbers, irrational numbers, and real numbers is covered. Finding the square roots is taught.

Lesson 12 GRAPHING LINEAR EQUATIONS

- Various relations are identified. Linear equations are graphed.

Lesson 13 FORMULAS

- Algebraic expressions and various formulas are evaluated. Rearranging of formulas is done to find particular variables.

Lesson 14 **PROBLEM SOLVING**

- Money and age problems are solved.

Lesson 15 MARKETING

- Various aspects of marketing such as profit, loss, discount, taxes, commission, simple interest, and compound interest are studied.

Lesson 16 PROBLEM TYPES

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Lesson 17 THE LANGUAGE OF ALGEBRA

- Various algebraic terminology is discussed. Terms such as variable, constant, equation, algebraic expression, term, factor, numerical and literal coefficients, polynomial, monomial, binomial, trinomial, degree and descending order are discussed.

Lesson 18 ADDING AND SUBTRACTING POLYNOMIALS

- Like and unlike terms are studied as well as inverses of polynomials. Addition and subtraction are applied to individual terms and then to larger polynomials.

Lesson 19 MULTIPLICATION AND DIVISION OF MONOMIALS

- Multiplication and division are done with monomials. Polynomials are multiplied and divided by monomials. The idea of Greatest Common Factor is applied to polynomials.

Lesson 20 REVIEW

- This lesson reviews the concepts taught in Lessons 1 to 19.

Lesson 21 SPECIAL PRODUCTS AND FACTORING

- Multiplying polynomials is studied further. Common factors are taken out of polynomials. The difference of squares method of factoring is covered.

Lesson 22 FACTORING

- Perfect squares and trinomials of the form $x^2 + bx + c$ are factored.

Lesson 23 POLYNOMIAL OPERATIONS

- Polynomials are added and subtracted, monomials are multiplied and divided, and binomials are multiplied. Factoring is reviewed.

Lesson 24 GEOMETRY

- A review is made of different types of angles and triangles. Segments and angles are constructed and bisected. Also, three different kinds of triangles are constructed.

Lesson 25 GEOMETRY

- Regular polygons are constructed. The Pythagorean Theorem and Heron's Formula are taught.

Lesson 26 PROPORTIONS

- Ratios are used to find distance, speed and time and to do scale drawings.

Lesson 27 MEASUREMENT

- Perimeter and area are reviewed. The area of regular polygons is found. Different types of prisms, pyramids, cones and cylinders are discussed.

Lesson 28 GEOMETRY

- The construction of models of prisms, pyramids and other regular polyhedra is taught. The surface area of prisms, pyramids, and cylinders is found.

Lesson 29 GEOMETRY

- The volume of prisms, cylinders, pyramids, and cones is determined.

Lesson 30 TEST

- Knowledge of the concepts taught in Lessons 1 to 29 is tested.

WELCOME TO MATHEMATICS 9

The Mathematics 9 course is designed to be used **with or without** the Holt Mathematics 3 textbook. The teaching notes in each lesson explain the ideas and concepts in detail. Should you desire additional information there are frequent references to Holt Mathematics 3.

General Instructions

1. This course consists of 30 lessons and a final examination.
2. You will need a compass, protractor, and a ruler marked in millimetres and centimetres.
3. Check the page numbers in each lesson to see that you course is complete. When you send in Lesson 1, ask for any pages that may be missing from your course, (List the lesson and the page number.)
4. Remove the staples from your course. Do not tear out the pages. Place the lessons in a loose-leaf binder.

Instructions for Doing Lessons

1. Begin work on Lesson 1 as soon as possible. Do the exercises and pages in proper order.
2. Read the teaching notes and study the examples that follow before answering questions in each section.

3. Do your work neatly in clearly visible pencil. Try to use an HB pencil.

4. If you find a question difficult, do as much of it as you are able; then tell your teacher what you do not understand. You may ask questions, if you wish, about parts that are not clear. Students in Alberta may call the Alberta Distance Learning Centre toll free by using one of the following methods.

   Method 1

   1. Look in your local telephone directory under the Government of Alberta for your local RITE number.

   2. Dial the RITE number.

   3. Ask for the Alberta Distance Learning Centre in Barrhead.

   NOTE: If there is no local RITE number use the following procedures.

   Method 2

   1. Dial the Operator (0).

   2. Ask for Zenith 22 333.

   3. Ask for the Alberta Distance Learning Centre in Barrhead.

5. The first section of each lesson is Keeping Skillful. It is a review of work you have already taken. You should be able to get all these questions correct.

6. A number of exercises have been prepared for you to correct yourself. After doing a Self-Correcting Exercise mark your own work using the solutions given. Study the questions you had wrong so you do not make the same errors in the assignment.

7. When you have completed a lesson, check your answers. Then submit your lesson promptly. Be sure to include a completed Lesson Record Form for each lesson that you have enclosed in the mailing envelope.

8. While you wait for a lesson to be returned, do the next lesson. Do not send in more than two mathematics lessons per week.

9. Lessons 10, 20 and 30 are review lessons. Review the previous lessons before doing these review lessons.

10. There is a final test for this course. Review all your lessons carefully before doing it. See the note about the final test which follows this introduction.

Corrected Lessons

1. When a corrected lesson is returned to you, review it carefully. Your teacher had made comments and corrections to help you avoid the same errors in future lessons.

2. If a lesson is returned to you with the word **Incomplete** or **Repeat** written on it, you must send back the entire lesson with any additional work requested by your teacher.

3. Place a graded lesson in your binder or rings. You will need it to review or refer to later.

4. Do not hesitate to ask questions about anything you do not understand. Your teachers are always ready to help you.

Lesson Gradings

Your standings on the lessons will be indicated by means of letter gradings on the following basis.

Letter Gradings	Range of Scale
A	80 – 100%
B	65 – 79%
C	50 – 64%
D	40 – 49%
F	0 – 30%

We wish you success and enjoyment in this course.

ADVANCE NOTICE: TESTING AND COURSE EVALUATION

To complete this course, you must write a supervised test set by the Alberta Distance Learning Centre before your registration expires. (Information about expiry dates is in the Handbook for Junior High Students Grades 7-9 you received with your application.) In order to pass this course you must score at least a D on the test and have a final mark of at least a C. (The test will make up 40% of your final mark. The other 60% will be based on course work, as evaluated by your correspondence teacher.) If you are dissatisfied with your mark, you may request to write an appeal paper. Such requests must be made within 30 days.

(a) **Classroom students** (taking any courses in an Alberta school):

If you are a classroom student taking correspondence courses, you do **not** need to submit an application for your final test. Test papers are sent automatically to the principal after you have completed your assignments satisfactorily.

Questions about the **scheduling** of final tests should be directed to your principal.

(b) **Non-classroom students** (studying exclusively by correspondence):

If you are studying exclusively by correspondence you will need to submit an application for your final test. A final test application form will be mailed to you when you are approaching the end of your program. You should return the completed application form promptly. The test will then be sent to your test supervisor.

Note: For the purpose of writing final tests, students who live outside Alberta come under the same regulations as those in category (b).

J.H.Gen. 5-88

A LESSON RECORD FORM MUST BE COMPLETED FOR EVERY LESSON SUBMITTED FOR CORRECTION, AS ILLUSTRATED BELOW

A Lesson Record form with the **correct** label attached **must** be enclosed with **every lesson** submitted for correction, as illustrated below.

Correct use of these labels will ensure prompt processing and grading of your lessons.

The enclosed **Lesson Labels** must be checked for spelling and address details.

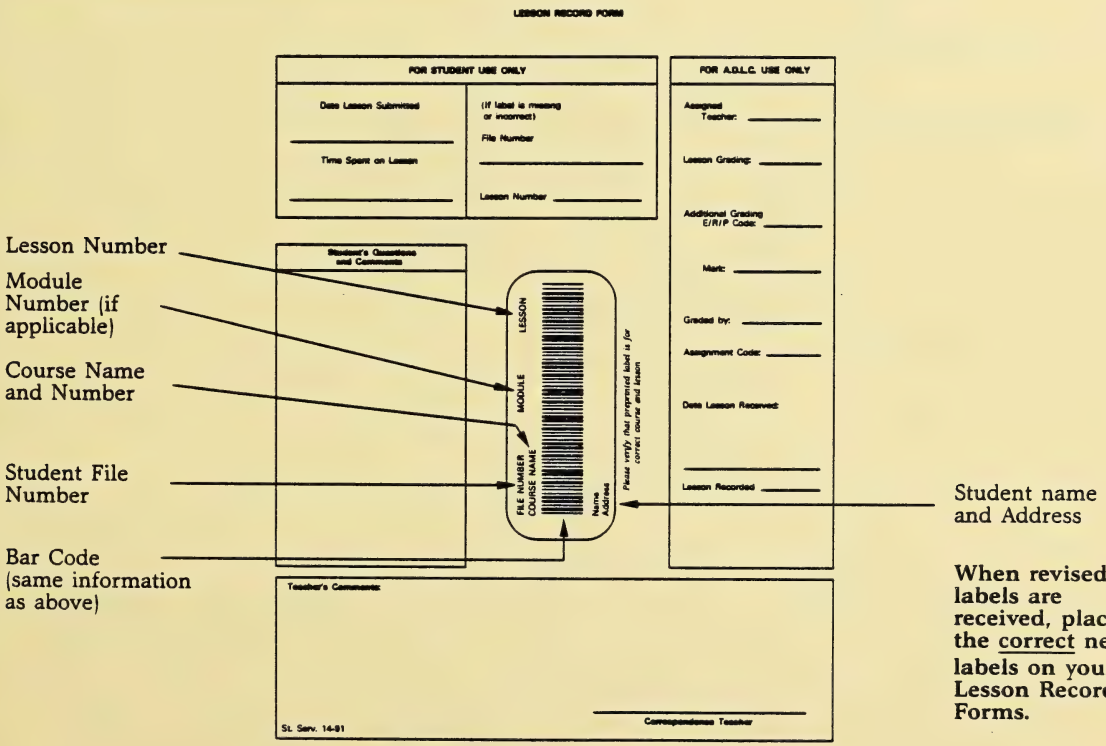
Please advise the Alberta Distance Learning Centre promptly of any changes in name, address, school, or any other details and we will issue a revised set of labels. Your file number is permanently assigned and **must** be included on all correspondence with the Alberta Distance Learning Centre. If the proper label and Lesson Record Form is not attached to each lesson as indicated it will delay your lessons being processed and credited to you.

Lesson labels are to be attached to the **lesson record forms** in the space provided for student name and address.

Check carefully to ensure that the **subject name, module number** and **lesson number** on each label corresponds exactly with the lesson you are submitting.

Labels are to be **peeled** off waxed backing paper and **stuck** on the lesson record form.

Only **one** label is to be placed on each lesson.



When revised labels are received, place the correct new labels on your Lesson Record Forms.

DO NOT MARK OR COVER BAR CODING.

CHANGE OF ADDRESS

If the address on your lesson record form differs from the address you supplied on your registration application, please explain. Indicate whether the different address is your home, school, temporary or permanent change of address.

LESSON RECORD FORM

0902 Math 9
Revised 11/90

FOR STUDENT USE ONLY

Date Lesson Submitted

_____

Time Spent on Lesson

_____

(If label is missing or incorrect)

File Number

_____

Lesson Number _____

FOR A.D.L.C. USE ONLY

Assigned Teacher: _____

Lesson Grading: _____

Additional Grading E/R/P Code: _____

Mark: _____

Graded by: _____

Assignment Code: _____

Date Lesson Received:

_____

Lesson Recorded _____

Student's Questions and Comments

Apply Lesson Label Here

Name

Address

Postal Code

Please verify that preprinted label is for correct course and lesson.

Teacher's Comments:

_____

Correspondence Teacher

St. Serv. 14-91

ALBERTA DISTANCE LEARNING CENTRE

MAILING INSTRUCTIONS FOR CORRESPONDENCE LESSONS

1. BEFORE MAILING YOUR LESSONS, PLEASE SEE THAT:

(1) All pages are numbered and in order, and no paper clips or staples are used.
(2) All exercises are completed. If not, explain why.
(3) Your work has been re-read to ensure accuracy in spelling and lesson details.
(4) The Lesson Record Form is filled out and the correct lesson label is attached.
(5) This mailing sheet is placed on the lesson.

2. POSTAGE REGULATIONS

Do **not** enclose letters with lessons.

Send all letters in a separate envelope.

3. POSTAGE RATES

First Class

Take your lesson to the Post Office and have it weighed. Attach sufficient postage and a green first-class sticker to the front of the envelope, and seal the envelope. Correspondence lessons will travel faster if first-class postage is used.

Try to mail each lesson as soon as it has been completed.

When you register for correspondence courses, you are expected to send lessons for correction regularly. Avoid sending more than two or three lessons in one subject at the same time.

St. Serv. 9-91

How to Succeed

You have shown that you are an achiever, that you are a rather exceptional person, as you have chosen to study by correspondence. You have discovered an alternative form of learning which many students feel is highly appropriate for them. You may believe as they do that correspondence study is closer to the preferred form of learning than traditional classroom instruction.

Now that you have indicated you want to learn by correspondence, we want to do all we can to help you succeed. Correspondence study has one thing in common with any other skill you might like to learn — from basketball to computer technology — it takes regular practice and guidance from a coach to help you achieve your goals. If you work on your course regularly, you will get the practice you need and, if you submit your lessons regularly, you will get any assistance you need from your teacher.

You will find the course work easier if you set aside some time each day for study and you attempt only part of the lesson at each session. Try to finish one lesson in the course each week and send the lesson for correction as soon as you have finished it.

If you follow these suggestions your correspondence teacher will have ample time to help you and you will be more likely to succeed in your studies. Start your course right away, today, and keep working on it until you reach your goal. You will find you will need determination and perseverance, but with support from your teacher and from your family and friends, you will be successful in your correspondence studies.

We at the Alberta Distance Learning Centre will do all we can to help you achieve your learning objective. Write to us or call us at 674-5333 if you need any special help with your course work.

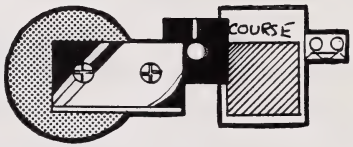
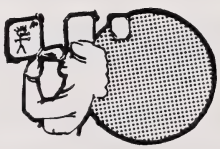
Garry Popowich
Director

Formula for Success in Correspondence Studies

If **X = Success**

then **X = a + b + c + d + e**

When

a =	Eager Student	
b =	High Quality Instructional Materials	
c =	Hard Work	
d =	Concerned Teacher	
e =	Regular Contact	

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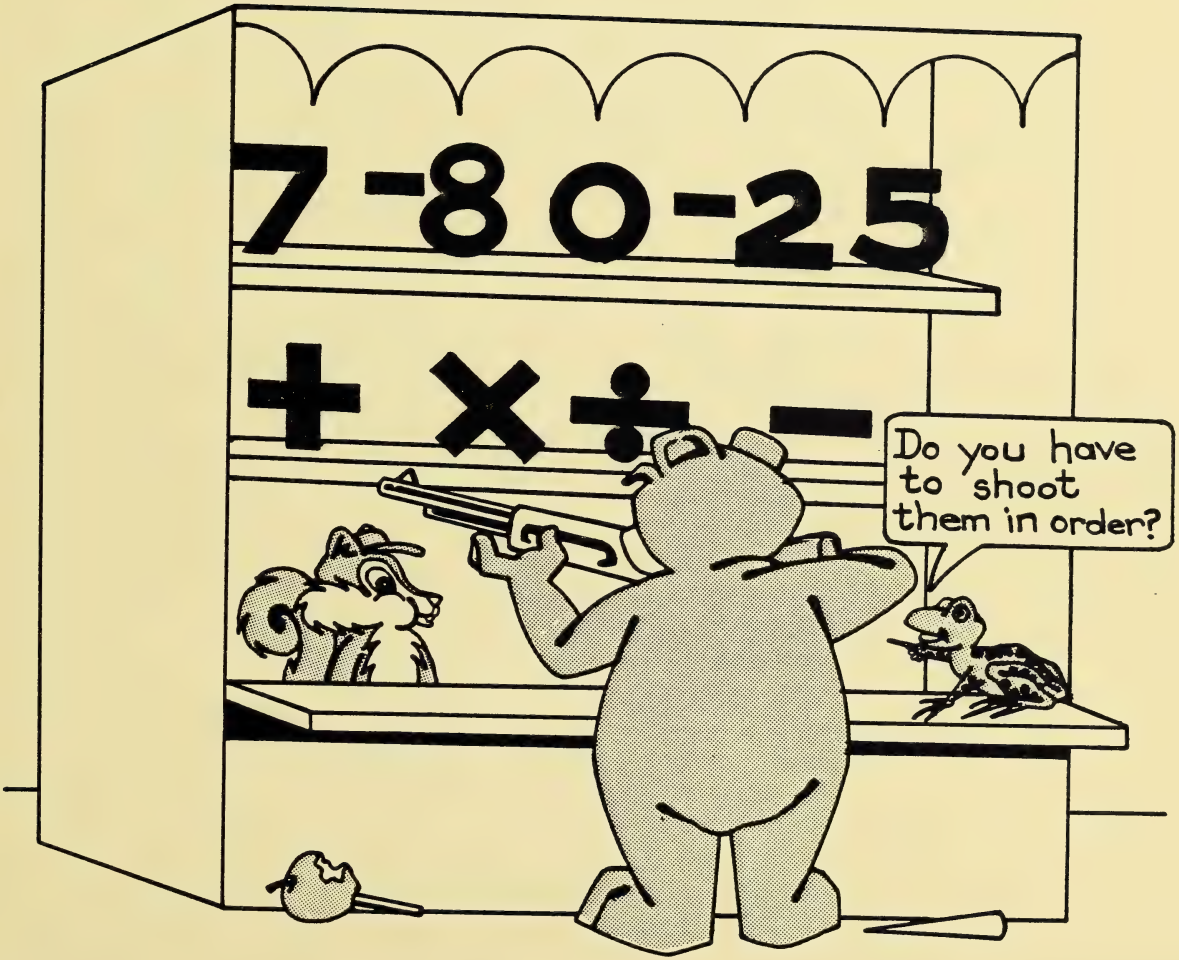
INTEGERS



In this lesson you will learn about:

- adding, subtracting, multiplying and dividing integers
- order of operations

KEEPING SKILLFUL

EXERCISE 1 Find the answers to the following questions. Be sure to show your work.

1. $\begin{array}{r} 486 \\ 9 \\ 88 \\ \underline{1053} \end{array}$

2. $\begin{array}{r} 89430 \\ \underline{-48876} \end{array}$

3. $\begin{array}{r} 48 \\ \underline{\times 39} \end{array}$

4. $\begin{array}{r} 657 \\ \underline{\times 978} \end{array}$

5. $89\overline{)17622}$

6. $192\overline{)39984}$

THE STUDY OF INTEGERS

For many situations throughout the day, whole numbers are commonly used to solve problems. However, at times opposite ideas are encountered. For example:

12°C above zero ⟷ 12°C below zero

500 m above sea level ⟷ 500 m below sea level

a gain of $20 ⟷ a loss of $20

For each of the examples above it may be said that they are equal in magnitude, that is, they have the same amount. Magnitude just tells you how far you are from point zero.

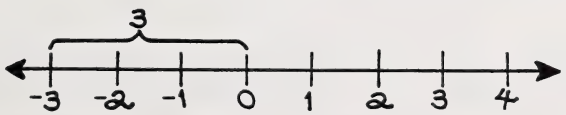
Another name for the magnitude of a number is absolute value.

Study the following:

The absolute value of -2 is 2	$\|-2\| = 2$
The absolute value of 2 is 2	$\|2\| = 2$
The absolute value of -5 is 5	$\|-5\| = 5$
The absolute value of 5 is 5	$\|5\| = 5$
The absolute value of 0 is 0	$\|0\| = 0$

The symbol for absolute value is | |. The absolute value of an integer helps you to understand the relative size of a number, without regard to its sign. It tells how far the graph of an integer is from the origin.

Example 1 What is the absolute value of -3?



-3 is 3 units from the origin. $|-3| = 3$

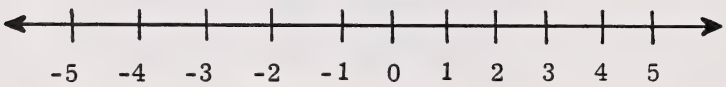
Note that the examples at the top of page 3 are not equal in direction. They are opposite in direction. To help us with the direction, positive and negative signs are used along with the magnitude.

12°C above zero can be shown as +12°C.
12°C below zero can be shown as -12°C.
500 m above sea level can be shown as +500 m.
500 m below sea level can be shown as -500 m.
A gain of $20 can be shown as +$20.
A loss of $20 can be shown as -$20.

> One set of numbers which has both magnitude and direction is the set of INTEGERS.

The set of Integers is represented by the letter I.

$I = \{. . . -2, -1, 0, 1, 2, . . .\}$



The numbers to the right of 0 are the positive integers. Remember that the positive sign (+) may be omitted.

The numbers to the left of 0 are the negative integers.

Zero is a number which may or may not have the positive or negative sign attached.

Reference Study pages 35 and 36 of *Holt Mathematics 3*.

EXERCISE 2 Give integers that could be used to represent the following.

1. (a) a loss of $60 _____ (b) a rise in temperature of 10°C _____

   (c) 100m above sea level _____

   (d) 25 m below sea level _____

2. Include the magnitude and the direction for the following questions. One is done for you.

(a) a loss of 13 m -13

(b) 1500 m above sea level _____

(c) a profit of $100 _____

(d) 4 floors down _____

(e) an increase of 12 km _____

(f) 8 runs behind _____

(g) 10 points ahead _____

PRINCIPLE OF OPPOSITES

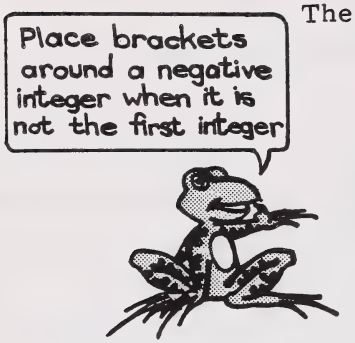
When you have temperatures above or below zero, or have a gain or loss, you are using the idea of opposites.

> +2 and -2 are also called opposites. The sum of +2 and -2 is 0.

The sum of two "opposites" is zero. Another name for opposite is additive inverse.

Reference Study pages 37 and 38 of *Holt Mathematics 3*.

ADDITION OF INTEGERS



The addition of integers involves knowing two basic rules.

1. If the two integers that you are adding have like signs, add their absolute values and include that sign.

   $$-5 + (-4) = -(5 + 4) = -9$$

2. If the two integers that you are adding have unlike signs, subtract their absolute values and include the sign of the integer with the largest magnitude.

   $+7 + (-2) = +5$

   Unlike signs.

   7 has the largest magnitude so the sign would be +.

Since positive integers are the same as whole numbers, the double positive signs are not necessary.

For example (+5) + (+4) will be written as 5 + 4. The brackets for cases like these will be left out.

However, include brackets around the negative integers for the time being.

The brackets are good to use as they separate the direction of the integer from the operation of addition.

Example 1 Find the value of 18 + (-4).

18 + (-4)	*18 is really +18. You have (+18) and (-4) or two integers with unlike signs, so you subtract.*
= 14	*Notice that the positive sign is not used. 14 is assumed to be +14.*

When you are asked to add more than two integers, it is a good practice to use the following procedure:

1. Group all the positive integers and add them.
2. Group all the negative integers and add them.
3. Use the rule for adding integers with unlike signs to find the sum of these positive and negative totals.

Example 2 Find the value of $14 + (-8) + 11$.

$14 + (-8) + 11 = \underbrace{14 + 11}_{\text{Group the positive integers.}} + \underbrace{(-8)}_{\text{Negative integer.}}$

$= 25 + (-8)$ ← Apply Rule 2.

$= 17$

Example 3 Find the value of $-20 + 18 + (-30) + 5$.

$-20 + 18 + (-30) + 5$

$= \underbrace{-20 + (-30)}_{\text{Negative integers.}} + \underbrace{18 + 5}_{\text{Positive integers.}}$

$= (-50) + 23$

You are now left with the sum of two integers with unlike signs. In order to finish this sum, subtract the absolute values. The sum is negative since 50 has a larger absolute value and is negative.

$= -27$

Remember that to add negative integers you add their absolute values and attach the minus sign to the result. Positive integers are added like whole numbers.

EXERCISE 3 Add the following integers.

1. $-10 + 4 =$

2. $4 + (-6) =$

3. $14 + (-3) =$

4. $-7 + (-6) =$

5. $-8 + 16 =$

6. $-4 + 5 + 6 =$

7. $-8 + (-8) + 8 =$

8. $17 + 25 + (-50) =$

9. $19 + (-20) + (-4) =$

10. $-9 + (-10) + (-11) =$

11. $-2 + (-18) + 20 + 6 =$

12. $-9 + 14 + (-17) + 4 =$

13. $68 + (-4) + (-5) + (-6) =$

SUBTRACTION OF INTEGERS

Reference Study page 40 of *Holt Mathematics 3.*

To subtract integers you will want to remember the rules for adding integers. Remember subtraction is the opposite of addition. To subtract an integer add its opposite.

> Two integers can be subtracted by changing the operation to addition and changing the sign of the second number.
>
> $6 - (+9) = 6 + (-9) = -3$

Example 1 Evaluate $16 - (+11)$.

$16 - (+11) = 16 + (-11)$ *Change operation to addition. Change 11 to -11.*

$= 16 - (+11)$ Now you have an addition question involving the sum of a positive number and a negative number. The rules for adding integers tell you to subtract their absolute values and attach the sign of the integer with the larger magitude.

$= 5$

Example 2 Evaluate $13 - (+17)$.

$13 - (+17) = 13 + (-17)$ *Change the operation.*

$= -4$ *The opposite of 17 is -17.*

Example 3 Evaluate $-20 - (-25)$.

$-20 - (-25) = -20 + 25$ *Change the operation.*

$= 5$ *The opposite of -25 is 25.*

SELF-CORRECTING EXERCISE A

Find the values of the following:

1. (a) $5 - 8$ (b) $-9 - 2$

(c) $8 - (-1)$ (d) $-4 - (-3)$

2. (a) $-16 - (-22)$ (b) $-30 - 14$

(c) $14 - 29$ (d) $38 - 42$

Check your answers with those on page 19 of this lesson. Correct any errors in your work.

When you encounter questions involving both addition and subtraction of integers, use the following procedure:

1. Change each subtraction to addition. (Remember to change the sign of the integer that follows the subtraction sign.)
2. Group all the positive integers and add them.
3. Group all the negative integers and add them.
4. Find the sum of these positive and negative totals.

Example 4 Evaluate $6 + 4 - (+2) + 3 - (-8) - (+4) + 9$.

$= 6 + 4 - (+2) + 3 - (-8) - (+4) + 9$ Change each subtraction operation to an addition operation and change the sign of the integer immediately after it.

$= 6 + 4 + (-2) + 3 + 8 + (-4) + 9$

Next group the five positive integers together and the two negative integers together.

$= \underbrace{6 + 4 + 3 + 8 + 9}_{\text{Positive integers.}} + \underbrace{(-2) + (-4)}_{\text{Negative integers.}}$

$= 30 + (-6)$ Find the sum of the positives and the sum of the negatives.

Sum of positives. *Sum of negatives.*

$= 24$ Finally, find the sum of the positive and negative totals.

SELF-CORRECTING EXERCISE B

Evaluate:

1. (a) $6 - (-3) + 4 - (-2)$ (b) $-2 - 2 - (-3) + (-6)$

2. (a) $17 - (-28) - 15 - 27$ (b) $-18 - 19 - 18 - (-32)$

Check your answers with those on page 19 on this lesson. Correct any errors in your work.

EXERCISE 4 Do the following questions.

1. $5 - (+9) =$

2. $-6 - (+4) =$

3. $17 - (-21) =$

4. $-18 - (-11) =$

5. $-3 + (-4) + 5 =$

6. $-11 + (-5) - (-16) =$

7. $25 + (-31) - 26 + 45 =$

8. $-14 - 17 - (-14) + (-20) =$

9. $-9 + (-9) + 27 - (-27) =$

10. $-234 - (-456) - (-183) =$

MULTIPLICATION OF INTEGERS

Reference Study pages 43 and 44 of *Holt Mathematics 3*.

When multiplying integers there are two basic rules to follow.

1. The product of two integers with like signs is positive.

$$+ \times + = +$$

$$- \times - = +$$

2. The product of two integers with unlike signs is negative.

$$- \times + = -$$

$$+ \times - = -$$

Example Multiply -4, -2 and -7.

$$-4 \times -2 \times -7 = (-4 \times -2) \times -7$$

Find the value within the brackets first.

$$= +8 \times -7$$

$$= -56$$

The product of an EVEN number of negative integers is a positive integer.

The product of an ODD number of negative integers is a negative integer.

EXERCISE 5 Do the following questions.

1. State whether the following products will be positive or negative.

   (a) -2×-2 *positive* (b) $-4 \times -3 \times +3$ _____

   (c) $-2 \times -2 \times -2 \times -2$ _____

   (d) $-2 \times -10 \times +2 \times -4$ _____

2. Find the product of each of the following. One is done for you.

   (a) $+12 \times (-8) =$

   (b) $-12 \times -14 \times 5 = (-12 \times -14) \times 5$
   $= +168 \times 5$
   $= 840$

   (c) $-25 \times +7 \times -4 =$

   (d) $-32 \times (-16) \times (-4) =$

Computations

$$\begin{array}{r} -12 \\ -14 \\ \hline 48 \\ 120 \\ \hline 168 \end{array} \qquad \begin{array}{r} 168 \\ 5 \\ \hline 840 \end{array}$$

DIVISION OF INTEGERS

Reference Study page 46 of *Holt Mathematics 3.*

The same rules that you studied for multiplication of integers apply to the division of integers. There are two basic rules to follow.

1. The quotient of two integers with like signs is positive.

$$+ \div + = +$$
$$- \div - = +$$

2. The quotient of two integers with unlike signs is negative.

$$- \div + = -$$
$$+ \div - = -$$

Example Solve $-64 \div 4 \div (-8)$

$$-64 \div 4 \div (-8) = (-64 \div 4) \div -8$$

Two unlike signs make the answer negative.

$$= -16 \div -8$$

Two like signs make the answer positive.

$$= 2$$

SELF-CORRECTING EXERCISE C

Divide.

1. (a) $\frac{132}{-12}$ (b) $\frac{-144}{-12}$

Check your answers with those on page 19 of this lesson. Correct any errors in your work.

EXERCISE 6 Find the quotients of each of the following.

(a) $\frac{-125}{5}$

(b) $-96 \div 6 \div 4$

(c) $56 \div (-2) \div (-7)$

(d) $-200 \div (-5) \div 8$

Computation

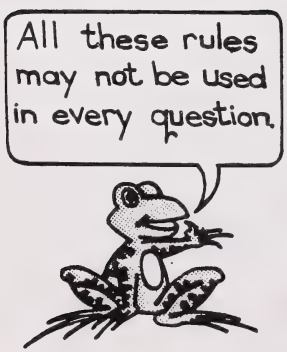
ORDER OF OPERATIONS

When working with integers, it may be necessary to use more than one operation to solve a problem. To find the correct solution, a certain order must be followed.

Reference

Study page 8 of *Holt Mathematics 3.* These are the rules to follow in placing the integers in order.

Rules



1. First perform the operation inside the brackets.
2. If the expression has a numerator and a denominator, simplify the numerator and denominator separately and then reduce the expression.
3. Then do the multiplication and division in the order that they occur, reading from left to right.
4. Finally, do the addition and subtraction in the order in which they occur, reading from left to right.

Example 1 Evaluate $-15 - [8 + (-5)] \div (-24 \div 8)$.

$-15 - [8 + (-5)] \div (-24 \div 8)$ *Do what is inside the brackets first.*

$= -15 - (+3) \div (-3)$ *Next do the division.*

$= -15 - (-1)$ *Finally, subtract.*

$= -15 + 1$

$= -14$

Example 2 Evaluate $\dfrac{15 \div (-3)}{2 \times (-5) + 5} + 16 - (-12)$.

$\dfrac{15 \div (-3)}{2 \times (-5) + 5} + 16 - (-12)$ *Since there are no operations inside the brackets, simplify the numerator and denominator.*

$= \dfrac{-5}{-10 + 5} + 16 - (-12)$

$= \dfrac{-5}{-5} + 16 - (-12)$ *Reduce the expression.*

$= 1 + 16 - (-12)$ *Add and subtract in the order which they occur.*

$= 17 - (-12)$

$= 17 + 12$

SELF-CORRECTING EXERCISE D

Simplify the following.

1. $\dfrac{3\times(9-2)+8\div4+2}{6\times(11-8)-(8+5)}$

Check your answers with those on page 19 of this lesson.
Correct any errors in your work.

EXERCISE 7

Part A Review the rules for order of operations and then do the following questions. Show your work.

1. $18 \times [\ 6 - (-3)] \div [\ 16 + (-13)] =$

2. $-24 - 8 \div 2 \times 3 =$

3. $\dfrac{15 \div (8 - 3) \times 3}{18 + (-7 - 6)} =$

4. $\dfrac{-2 \times 3 + 4 \times (-3)}{5 \times (-4) - 33 \div -3} =$

KEY TO SELF-CORRECTING EXERCISES IN LESSON 1

SELF-CORRECTING EXERCISE A, page 10.

1. (a) $5-8=5+-8=-3$ (b) $-9-2=-9+-2=-11$
   (c) $8--1=8++1=+9$ (d) $-4-(-3)=-4++3=-1$

2. (a) $-16-(-22)=-16++22=+6$ (b) $-30-14=-30+-14=-44$
   (c) $14-29=14+-29=-15$ (d) $38-42=38+-42=-4$

SELF-CORRECTING EXERCISE B, page 11.

1. (a)
$$\begin{aligned} &6-(-3)+4-(-2)\\ &=6+3+4+2\\ &=15 \end{aligned}$$

(b)
$$\begin{aligned} &-2-2-(-3)+(-6)\\ &=-2+(-2)+3+(-6)\\ &=3+(-2)+(-2)+(-6)\\ &=3+(-10)\\ &=-7 \end{aligned}$$

2. (a)
$$\begin{aligned} &17-(-28)-15+(-27)\\ &=17+28+(-15)+(-27)\\ &=45+(-42)\\ &=3 \end{aligned}$$

(b)
$$\begin{aligned} &-18-19+(-18)-(32)\\ &=-18+(-19)+(-18)+32\\ &=32+(-18)+(-19)+(-18)\\ &=32+(-55)\\ &=-23 \end{aligned}$$

SELF-CORRECTING EXERCISE C

1. (a) $\dfrac{132}{-12}=-11$

   (b) $\dfrac{-144}{-12}=+12$

SELF-CORRECTING EXERCISE D, page 17

1.
$$\begin{aligned} &\frac{3\times(9-2)+8\div4+2}{6\times(11-8)-(8+5)}\\ &=\frac{\overbrace{3\times7}+\overbrace{8\div4}+2}{\underbrace{6\times3}-13}\\ &=\frac{21+2+2}{18-13}\\ &=\frac{25}{5}\\ &=5 \end{aligned}$$

END OF LESSON 1

JUNIOR HIGH MATHEMATICS QUESTIONNAIRE
Math 9 (Holt)

Name: _____

Address: _____

_____

_____

File #: _____ Telephone No. _____

Birthdate: Year: _____ Month: _____ Day: _____

1. Details of last mathematics course completed successfully:

   Course: _____ Year Completed: _____

   School: _____ Final Mark: _____

2. What school are you attending now (if any)? _____

3. Are there other students in your school or district taking correspondence courses? _____

4. Do you study with any of them? _____

5. What other courses are you taking by correspondence? _____

   _____

   _____

6. Have you taken correspondence before? When? _____

   Which course(s) did you study? _____

   _____

7. Why are you taking correspondence now? _____

   _____

8. Mention any special factors (handicaps, illness, family problems, etc) which may influence your progress in your studies.

   _____

   _____

   _____

   _____

In order to get to know you better, we would like you to tell us a little bit about yourself. Write a paragraph or two telling us about yourself, and your plans for the future, or anything else that you think is interesting.

LESSON RECORD FORM

0902 Math 9
Revised 11/90

FOR STUDENT USE ONLY

Date Lesson Submitted

_____

Time Spent on Lesson

_____

(If label is missing or incorrect)

File Number

_____

Lesson Number _____

FOR A.D.L.C. USE ONLY

Assigned Teacher: _____

Lesson Grading: _____

Additional Grading E/R/P Code: _____

Mark: _____

Graded by: _____

Assignment Code: _____

Date Lesson Received:

_____

Lesson Recorded _____

Student's Questions and Comments

Apply Lesson Label Here

Name

Address

Postal Code

Please verify that preprinted label is for correct course and lesson.

Teacher's Comments:

_____

Correspondence Teacher

St. Serv. 14-91

ALBERTA DISTANCE LEARNING CENTRE

MAILING INSTRUCTIONS FOR CORRESPONDENCE LESSONS

1. BEFORE MAILING YOUR LESSONS, PLEASE SEE THAT:

(1) All pages are numbered and in order, and no paper clips or staples are used.
(2) All exercises are completed. If not, explain why.
(3) Your work has been re-read to ensure accuracy in spelling and lesson details.
(4) The Lesson Record Form is filled out and the correct lesson label is attached.
(5) This mailing sheet is placed on the lesson.

2. POSTAGE REGULATIONS

Do **not** enclose letters with lessons.

Send all letters in a separate envelope.

3. POSTAGE RATES

First Class

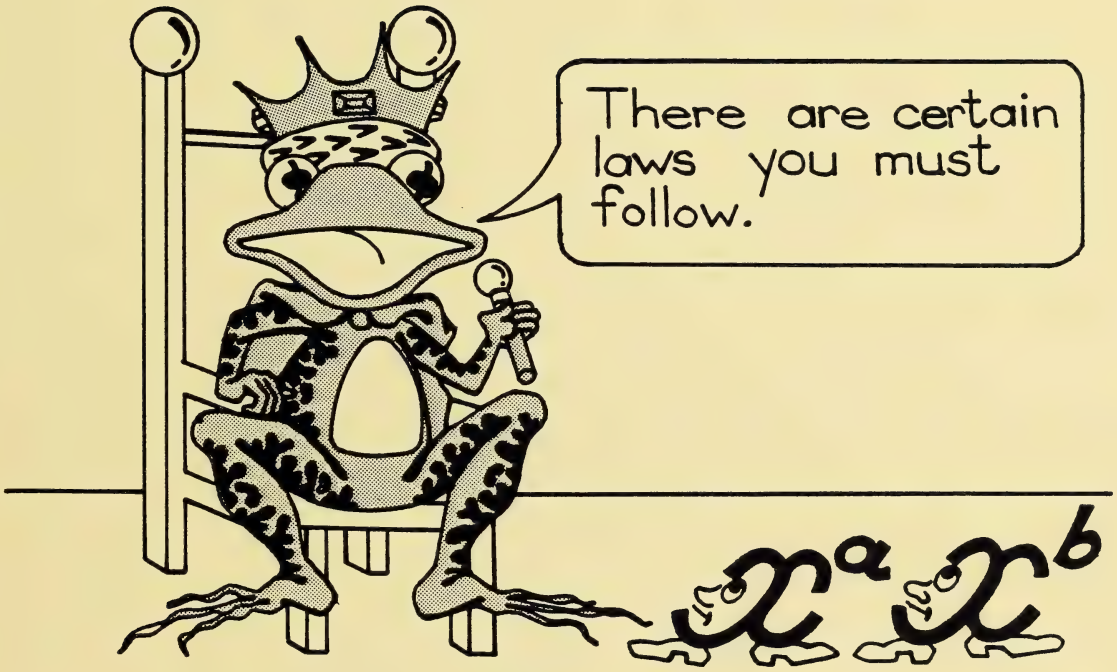
Take your lesson to the Post Office and have it weighed. Attach sufficient postage and a green first-class sticker to the front of the envelope, and seal the envelope. Correspondence lessons will travel faster if first-class postage is used.

Try to mail each lesson as soon as it has been completed.

When you register for correspondence courses, you are expected to send lessons for correction regularly. Avoid sending more than two or three lessons in one subject at the same time.

St. Serv. 9-91

POWERS



In this lesson you will learn about:

- evaluating positive and negative powers
- using the following laws:

$$x^a \cdot x^b = x^{a+b}$$

$$x^a \div x^b = x^{a-b}$$

$$(x^a)^b = x^{a \times b}$$

$$x^{-a} = \frac{1}{x^a}$$

KEEPING SKILLFUL

EXERCISE 1 1. Perform the following operations.

(a) $-6 + 4 =$

(b) $-98 + (-13) =$

(c) $5 - 24 =$

(d) $-11 - (-32) =$

(e) $16(-2) =$

(f) $(-8)(-61) =$

(g) $-16 \div 2 =$

(h) $-144 \div -6 =$

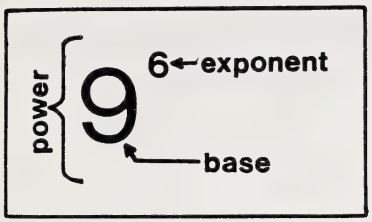
2. Evaluate the following. Remember to use the rules for order of operations.

(a) $-3 \times 9 + 42 \div -6 + 1 \times -4$

(b) $[2(8 + 3)] \div 10 + 1$

EXPONENTIAL NOTATION

Reference Study page 52 of *Holt Mathematics 3.*



base - used as a factor.

exponent - indicates the number of times the base is used as a factor.

power - the whole expression.

There are several ways to read 9^6. Some of these ways are:

9 to the power of 6, or

9 to the sixth, or

the sixth power of 9.

To evaluate 9^6, the base 9 must be used 6 times as a factor.

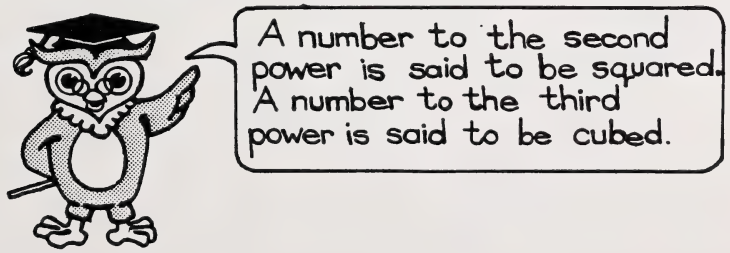
$$9^6 = 9 \times 9 \times 9 \times 9 \times 9 \times 9$$

$$= (9 \times 9) \times (9 \times 9) \times (9 \times 9)$$ *Grouping the factors in pairs makes the calculations easier.*

$$= 81 \times 81 \times 81$$

$$= 531\,441$$

Note Notice that 9^6 does not equal 9×6.



EXERCISE 2

1. Write these as powers.

(a) $5 \times 5 \times 5 \times 5 =$ _____

(b) $(-8) \times (-8) \times (-8) =$ _____

(c) $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 =$ _____

(d) $(-7)\ (-7)\ (-7)\ (-7)\ (-7)\ (-7) =$ _____

(e) $-4 =$ _____

(f) $10 =$ _____

2. Evaluate the following:

(a) $4^3 =$ _____ = _____

(b) $7^2 =$ _____ = _____

(c) $4^5 =$ _____ = _____

(d) $10^3 =$ _____ = _____

3. Write in words how you would read the following.

(a) 5^7

(b) 4^2

(c) 8^3

POWERS INVOLVING NEGATIVE NUMBERS AND BRACKETS

It is important to realize that $(-5)^2$ is not the same as -5^2.

The brackets used in $(-5)^2$ indicate that the negative sign and the 5 are grouped together to form the base. Thus, the -5 is squared.

$(-5)^2 = (-5) \times (-5)$
$= +25$

$(-5)^2$ means the square of -5.

Now consider -5^2. There are no brackets to indicate that the negative sign is to be grouped with the 5. Therefore, only the 5 is squared.

$-5^2 = -(5 \times 5)$
$= -25$

-5^2 means the negative value of the square of five.

In this course you will sometimes see powers such as -5^2 written as (-5^2) or $-(5)^2$. In each of these cases the negative sign and the 5 are not grouped together. So again, it is just the 5 that is being squared.

$(-5^2) = -(5^2)$
$= -(5 \times 5)$
$= -25$

Since the negative sign is not to be squared, move it outside the brackets.

$-(5)^2 = -(5 \times 5)$
$= -25$

Also you will see questions such as $-(-4^2)$ and $-(-4)^2$. You may use square brackets to simplify these.

$-(-4^2) =$
$-[-(4 \times 4)] =$
$-[-16] =$
$+16$

Since the negative sign is not be squared, move it outside the brackets.

$-(-4)^2 =$
$-[(-4)(-4)] =$
$-[16] =$
-16

Notice that the negative sign is squared.

SELF-CORRECTING EXERCISE A

Write these as the product of factors and evaluate.

1. (a) 3^4 (b) $(-8)^3$

   (c) 5^2

   (d) $(-6)^2$ (e) $(-2)^4$

2. (a) 4^4 (b) 5^3

   (c) 6^3 (d) 10^1

3. (a) $\left(-4^2\right)$ (d) $(-5)^3$

   (b) $(-4)^2$ (e) $(-6)^4$

   (c) $\left(-5^3\right)$ (f) $\left(-6^4\right)$

Check your answers with those on page 23 of this lesson.
Correct any errors in your work.

EXERCISE 3 Evaluate the following.

1. $(-2)^4$

2. (-2^4)

3. -7^2

4. $(-7)^2$

5. (-4^3)

6. $-(-4^3)$

7. $-(-5^3)$

8. $-(-5)^3$

9. $-(-3^2)$

10. $-(-3)^2$

EVALUATING EXPRESSIONS WITH EXPONENTS

Reference

Study Examples 1 and 2 on page 53 of *Holt Mathematics 3*.

On page 4 of this lesson you learned that the location of brackets and exponents is very important. Study the following examples.

Example 1

Evaluate $[5(3)]^2$

The square brackets indicate that the 5 and 3 are grouped together as the base of this power.

Thus, $[5(3)]^2$ means the square of $5(3)$.

$[5(3)]^2 = 5(3) \times 5(3)$

$= 15 \times 15$

$= 225$

Recall that these curved brackets indicate multiplication.

Example 2

Evaluate $5(3)^2$

Notice that there are no brackets grouping the 5 and 3 together as the base.

Thus, $5(3)^2$ means 5 times the square of 3.

$5(3)^2 = 5 \times 3^2$

$= 5 \times 9$

$= 45$

From the above examples you can see that $[5(3)]^2$ and $5(3)^2$ are not equal.

In Lesson 1 you learned the rules of operations. The rules have been rewritten here to include powers.

Rules of Order of Operations

1. First do the operations that are within brackets.
2. Then evaluate the powers.
3. Then do the multiplication and division as they occur (going from left to right).
4. Then do the addition and subtraction as they occur (going from left to right).

Example 3

Evaluate $(-9^2)(-3)^2$

$$(-9^2)(-3)^2 = (-9^2) \times (-3)^2$$

$$= -(9 \times 9) \times (-3 \times -3) \quad \textit{Evaluate each power.}$$

$$= -81 \times +9 \quad \textit{Then, multiply.}$$

$$= -729$$

Example 4

Evaluate $4^2 + (-2)^3$

$$4^2 + (-2)^3 = (4 \times 4) + (-2 \times -2 \times -2) \quad \textit{Evaluate each power.}$$

$$= 16 + -8 \quad \textit{Then, add.}$$

$$= 8$$

SELF-CORRECTING EXERCISE B

Evaluate each of the following.

1. $4(6)^2$

2. $[4(6)]^2$

3. $[3(2)]^3$

4. $3(2)^3$

Check your answers with those on page 23 of this lesson. Correct any errors in your work.

EXERCISE 4 Evaluate the following. Use the rules of order of operations.

1. $(-4)(-3)^2$

2. $[(-4)(-3)]^2$

3. $(-5^3)(2^2)$

4. $-(7)^2(3)^3$

5. $4^3 + 3^2$

6. $3^4 - 2^4$

ALGEBRA EXPRESSIONS

Reference

Study Example 1 on page 54 of *Holt Mathematics 3*.

An example of an algebraic expression is $5x^3$. The value of the expression $5x^3$ is dependent on the number used to replace the variable.

Example

Evaluate $5x^3$ if $x = -3$

$5x^3 = 5 \times x^3$

$= 5 \times (-3)^3$ *Replace the variable with -3.*

$= 5 \times (-3 \times -3 \times -3)$ *Evaluate the power.*

$= 5 \times -27$ *Multiply.*

$= -135$

EXERCISE 5 Evaluate the following.

1. x^3, if $x = -5$.

2. a^5, if $a = -2$.

3. $5y^2$, if $y = -4$.

4. $-3b^4$, if $b = 3$.

5. $(3m)^2$, if $m = 4$.

6. $-4(p^3)$, if $p = 10$.

THE FIRST POWER AND ZERO POWER

Study the following:

$3^5 = 3 \times 3 \times 3 \times 3 \times 3 = 243$

$3^4 = 3 \times 3 \times 3 \times 3 = 81$

$3^3 = 3 \times 3 \times 3 = 27$

$3^2 = 3 \times 3 = 9$

$3^1 = 3 = 3$

$3^0 = = 1$

The answers 243, 81, 27, 9, 3 and 1 can each be obtained by dividing the next higher power by 3. By following this pattern it is logical that $3^1 = 3$ and $3^0 = 1$.

First Power

> Any base to the first power equals the base itself.
>
> $9^1 = 9$; $(-5)^1 = -5$.

Note

Any number or variable without an exponent is considered to be to the first power. For example, $5 = 5^1$ and $y = y^1$.

Zero Power

> Any base to zero power equals one.
>
> $6^0 = 1$; $(-12)^0 = 1$

Note

No meaning is given to the symbol 0^0; that is, it is **undefined.**

SELF-CORRECTING EXERCISE C

Evaluate the following.

1. 19^1 = _____

2. 21^0 = _____

3. 0^0 = _____

4. 0^1 = _____

5. $(-39)^1$ = _____

6. $(-84)^0$ = _____

Check your answers with those on page 23 of this lesson. Correct any errors in your work.

EXERCISE 6 Evaluate the following.

1. 17^1 = _____

2. 17^0 = _____

3. $(-21)^1$ = _____

4. $(-21)^0$ = _____

5. 0^0 = _____

6. 0^1 = _____

NEGATIVE EXPONENTS

On page 12 of this lesson some of the powers of 3 were evaluated. Each power of 3 could be determined by dividing its successive higher power by 3. This pattern can be continued.

$3^3 = 27$

$3^2 = 9$ $\quad (27 \div 3 = 9)$

$3^1 = 3$ $\quad (9 \div 3 = 3)$

$3^0 = 1$ $\quad (3 \div 3 = 1)$

$3^{-1} = \frac{1}{3}$ $\quad (1 \div 3 = \frac{1}{3})$

$3^{-2} = \frac{1}{9}$ $\quad (\frac{1}{3} \div 3 = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9})$

$3^{-3} = \frac{1}{27}$ $\quad (\frac{1}{9} \div 3 = \frac{1}{9} \times \frac{1}{3} = \frac{1}{27})$

Notice

$3^{-1} = \frac{1}{3}$ or $\frac{1}{3^1}$

$3^{-2} = \frac{1}{9}$ or $\frac{1}{3^2}$

$3^{-3} = \frac{1}{27}$ or $\frac{1}{3^3}$

Negative Exponent Law

Any base to a negative power is the same as one over that base to an equivalent positive power.

$3^{-4} = \frac{1}{3^4}$, $\quad (-5)^{-3} = \frac{1}{(-5)^3}$

Example 1 Evaluate 9^{-3}.

$$9^{-3} = \frac{1}{9^3}$$

$$= \frac{1}{9 \times 9 \times 9}$$

$$= \frac{1}{729}$$

Example 2 Evaluate $(-3)^{-3}$

$$(-3)^{-3} = \frac{1}{(-3)^3}$$

$$= \frac{1}{-3 \times -3 \times -3}$$

$$= \frac{1}{-27}$$

$$= -\frac{1}{27}$$

Reference Study page 60 of *Holt Mathematics 3*. Page 60 proves once again that $x^{-\alpha}$ equals $\frac{1}{x^{\alpha}}$, if $x \neq 0$.

EXERCISE 7 Evaluate the following.

1. $5^{-2} =$

2. $7^{-3} =$

3. $2^{-5} =$

4. $4^{-4} =$

5. $(-2)^{-3} =$

6. $(-5)^{-4} =$

PRODUCT OF POWERS

Study the following

$4^3 \times 4^2 = (4 \times 4 \times 4)(4 \times 4)$

$= 4 \times 4 \times 4 \times 4 \times 4$

$= 4^5$

The same result could be obtained by adding the exponents:

$4^3 \times 4^2 = 4^{3+2} = 4^5$

Product of Powers Law

To multiply powers of the same base, add their exponents.

$5^8 \times 5^7 = 5^{8+7} = 5^{15}$,

$(-3)^2(-3)^4 = (-3)^{2+4} = (-3)^6$

SELF-CORRECTING D

Using the Product of Powers Law, express each of the following as a power.

1. $(3^5)(3^2) =$

2. $(4^2)(4^7)$

3. $(2^4)(2) =$

4. $(-2)^2(-2)^3$

Check your answers with those on page 24 of this lesson. Correct any errors in your work.

EXERCISE 8

Using the Product of Powers Law, express each of the following as a power.

1. $(3^2)(3^2)$

2. $(5)(5^2)$

3. $(-4)^2(-4)^5$

4. $(8^6)(8^4)$

5. $(-2)^9(-2)^2$

6. $(-4)(-4)^3$

QUOTIENT OF POWERS

Study the following:

$$6^5 \div 6^2 = \frac{6^5}{6^2}$$

$$= \frac{6 \times 6 \times 6 \times 6 \times 6}{6 \times 6}$$

$$= 6 \times 6 \times 6$$

$$= 6^3$$

The same result could be obtained by subtracting the exponents.

$$6^5 \div 6^2 = 6^{5-2} = 6^3$$

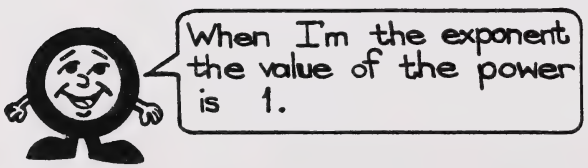
Quotient of Powers Law

To divide powers of the same base, subtract their exponents.

$$8^5 \div 8^2 = 8^{5-2} = 8^3,$$

$$(-4)^7 \div (-4)^2 = (-4)^{7-2} - (-4)^5$$

Note $5^3 \div 5^3 = 3^{3-3} = 5^0 = 1$



SELF-CORRECTING EXERCISE E

Using the Quotent of Powers Law, express each of the following as a power.

1. $8^5 \div 8^3$

2. $4^7 \div 4^4$

3. $(-9)^8 \div (-9)^2$

4. $8^3 \div 8^3$

5. $9^7 \div 9$

6. $(-8)^5 \div (-8)^5$

Check your answers with those on page 24 of this lesson. Correct any errors in your work.

EXERCISE 9

Using the Quotient of Powers Law, express each of the following as a power.

1. $4^5 \div 4^2$

2. $5^6 \div 5^3$

3. $(-8)^6 \div (-8)^4$

4. $7^3 \div 7^5$

5. $4^8 \div 4^8$

6. $2^9 \div 2$

7. $3^5 \div 3$

8. $(-4)^3 \div (-4)$

THE POWER OF A POWER LAW

Study the following:

$$(5^2)^3 = 5^2 \times 5^2 \times 5^2$$
$$= (5 \times 5)(5 \times 5)(5 \times 5)$$
$$= 5 \times 5 \times 5 \times 5 \times 5 \times 5$$
$$= 5^6$$

The same result can be obtained by multiplying the exponents:

$$(5^2)^3 = 5^{2 \times 3} = 5^6$$

Power of a Power Law

To find a power of a given power, multiply the exponents.

$$(5^3)^2 = 5^{3 \times 2} = 5^6,$$

$$[(-3)^3]^4 = (-3)^{3 \times 4} = (-3)^{12}$$

SELF-CORRECTING EXERCISE F

Using the Power of a Power Law, express each of the following as power.

1. $(2^3)^5$

2. $(3^2)^4$

3. $[(-2)^3]^8$

4. $[(-5)^2]^4$

Check your answer with those on page 24 of this lesson. Correct any errors in your work.

EXERCISE 10

Using the Power of a Power Law, express each of the following as a power.

1. $(3^2)^4$

2. $[(-2)^3]^5$

3. $(5^0)^4$

4. $[(-3)^4]^0$

5. $(5^{-2})^3$

6. $(8^{-3})^{-5}$

EXERCISE 11

Place the letter of the best answer for each of the following in the blank provided.

_____ 1. 4^3 is equal to

(a) 64 (b) 81 (c) 12 (d) $\frac{3}{4}$

_____ 2. -2^4 is equal to

(a) -16 (b) -8 (c) $\frac{-2}{4}$ (d) +16

_____ 3. $(-5)^2$ is equal to

(a) -25 (b) $\frac{1}{25}$ (c) $\frac{-2}{25}$ (d) 25

_____ 4. 5^0 is equal to

(a) 5 (b) 1 (c) 0 (d) -5

_____ 5. 3^1 is equal to

(a) 3 (b) 1 (c) 0 (d) -3

Summary

1. Any base to the first power equals the base itself.

$$9^1 = 9, \quad (-5)^1 = -5$$

2. Any base to the zero power equal one.

$$6^0 = 1, \quad (-12)^0 = 1$$

3. Any base to a negative power is the same as one over that base to an equivalent positive power.

$$3^{-2} = \frac{1}{3^2}, \quad (-5)^{-3} = \frac{1}{(-5)^3}$$

4. To multiply powers of the same base, add their exponents.

$$5^8 \times 5^7 = 5^{8+7} = 5^{15}$$

$$(-3)^2(-3)^4 = (-3)^{2+4} = (-3)^6$$

5. To divide powers of the same base, subtract their exponents.

$$8^5 \div 8^2 = 8^{5-2} = 8^3$$

$$(-4)^7 \div (-4)^2 = (-4)^{7-2} = (-4)^5$$

6. To find a power of a given power, multiply the exponents.

$$(5^3)^2 = 5^{3 \times 2} = 5^6$$

$$[(-3)^3]^4 = (-3)^{3 \times 4} = (-3)^{12}$$

KEY TO SELF-CORRECTING EXERCISES IN LESSON 2

SELF-CORRECTING EXERCISE A, page 6.

1. (a) 81 (b) -512 (c) 25 (d) 36 (e) 16
2. (a) 256 (b) 125 (c) 216 (d) 10
3. (a) $-(4\times4)=-16$ (b) $-4\times-4=+16$ (c) $-(5\times5\times5)=-125$
   (d) $-5\times-5\times-5=-125$ (e) $-6\times-6\times-6\times-6=1296$ (f) $-(6\times6\times6\times6)=-1296$

SELF-CORRECTING EXERCISE B, page 10.

1. $4(6)^2 = 4 \times 6^2$
   $= 4 \times 36$
   $= 144$

   Evaluate the power before you multiply.

2. $[4(6)]^2 = 24^2$
   $= 24 \times 24$
   $= 576$

   Do the operations that are within the brackets first.

3. $[3(2)]^3 = 6^3$
   $= 6 \times 6 \times 6$
   $= 36 \times 6$
   $= 216$

   Do the operations that are within the brackets first.

4. $3(2)^3 = 3 \times (2)^3$
   $= 3 \times 2 \times 2 \times 2$
   $= 3 \times 8$
   $= 24$

   Evaluate the power before you multiply.

SELF-CORRECTING EXERCISE C, page 13.

1. $19^1 = \underline{19}$
2. $21^0 = \underline{1}$
5. $(-39)^1 = \underline{-39}$
6. $(-84)^0 = \underline{1}$
3. 0^0 is not defined
4. $0^1 = 0$

SELF-CORRECTING EXERCISE D, page 17.

1. $(3^5)(3^2) = 3^{5+2} = 3^7$

2. $(4^2)(4^7) = 4^{2+7} = 4^9$

3. $(2^4)(2) = 2^{4+1} = 2^5$

4. $(-2)^2(-2)^3 = (-2)^{2+3} = (-2)^5$

SELF-CORRECTING EXERCISE E, page 19.

1. $8^5 \div 8^3 = 8^{5-3} = 8^2$

2. $4^7 \div 4^4 = 4^{7-4} = 4^3$

3. $(-9)^8 \div (-9)^2 = (-9)^{8-2} = (-9)^6$

4. $8^3 \div 8^3 = 8^{3-3} = 8^0 = 1$

5. $9^7 \div 9 = 9^{7-1} = 9^6$

6. $(-8)^5 \div (-8)^5 = (-8)^{5-5} = (-8)^0 = 1$

SELF-CORRECTING EXERCISE F, page 20.

1. $(2^3)^5 = 2^{3\times5} = 2^{15}$

2. $(3^2)^4 = 3^{2\times4} = 3^8$

3. $[(-2)^3]^8 = (-2)^{3\times8} = (-2)^{24}$

4. $[(-5)^2]^4 = (-5)^{2\times4} = (-5)^8$

END OF LESSON 2

LESSON RECORD FORM

0902 Math 9
Revised 11/90

FOR STUDENT USE ONLY

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Time Spent on Lesson

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(If label is missing or incorrect)

File Number

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Lesson Number _____

Student's Questions and Comments

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ALBERTA DISTANCE LEARNING CENTRE

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1. BEFORE MAILING YOUR LESSONS, PLEASE SEE THAT:

(1) All pages are numbered and in order, and no paper clips or staples are used.
(2) All exercises are completed. If not, explain why.
(3) Your work has been re-read to ensure accuracy in spelling and lesson details.
(4) The Lesson Record Form is filled out and the correct lesson label is attached.
(5) This mailing sheet is placed on the lesson.

2. POSTAGE REGULATIONS

Do **not** enclose letters with lessons.

Send all letters in a separate envelope.

3. POSTAGE RATES

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Take your lesson to the Post Office and have it weighed. Attach sufficient postage and a green first-class sticker to the front of the envelope, and seal the envelope. Correspondence lessons will travel faster if first-class postage is used.

Try to mail each lesson as soon as it has been completed.

When you register for correspondence courses, you are expected to send lessons for correction regularly. Avoid sending more than two or three lessons in one subject at the same time.

St. Serv. 9-91

FRACTIONS and RATIONAL NUMBERS

In this lesson you will learn about:

- calculating the GCF and LCM
- changing improper fractions to mixed numbers and vice versa
- adding and subtracting fractions, mixed numbers and rational numbers

KEEPING SKILLFUL

Exercise 1

1. Perform the following operations. Show all steps.

(a) $-32 - 8 =$ (b) $11 - (-42) =$

(c) $16 \times -4 =$ (d) $-140 \div -20 =$

(e) $-8 + 15 \times -2 \div (-7 + 5) =$

2. Evaluate the following. Show all steps.

(a) $5^4 =$ (b) $6^{-3} =$

(c) $(-3)^2 =$ (d) $-2^5 =$

(e) $(2^2)^3 =$ (f) $(5^1)^2 =$

(g) $4^3 \times 4^{-4} =$ (h) $16^{-8} \div 16^{-6} =$

FRACTIONS

Reference Study column one on page 6 and column one on page 7 of *Holt Mathematics 3.*

EXERCISE 2

1. List three examples of each of the following:

(a) Proper Fraction

_____

_____

(b) Improper Fraction

_____

_____

(c) Mixed Fraction

_____

_____

2. For the fraction, $\frac{a}{b}$, explain why $b \neq 0$.

_____

_____

CHANGING IMPROPER FRACTIONS TO MIXED NUMBERS

Recall that a fraction, $\frac{a}{b}$, is another way of showing division. $\frac{a}{b}$ means the same as $a \div b$.

Rule

> To change an improper fraction to a mixed number, divide the denominator into the numerator.

Example Write $\frac{10}{3}$ as a mixed number.

$$\frac{10}{3} = 3\frac{1}{3}$$

$$3\overline{)10}$$

The quotient is the whole number.

$$\underline{9}$$

$$1$$

The remainder put over the divisor, is the fraction.

Note A proper fraction can not be changed to a mixed number. For example, $\frac{7}{15}$ can not be written as a mixed number.

CHANGING MIXED NUMBERS TO IMPROPER FRACTIONS

In the previous section, $\frac{10}{3}$ was changed to $3\frac{1}{3}$. To do this operation in reverse, follow the rule below.

Shortcut

> To change a mixed number to an improper fraction, multiply the denominator and the whole number, then add on the numerator.

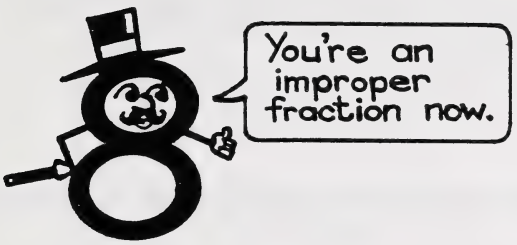
$$3\frac{1}{3} = \frac{9+1}{3} = \frac{10}{3}$$

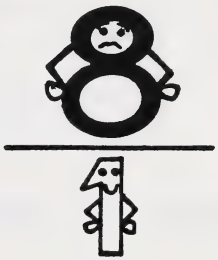
(+ over the 1, × over the 3)

Note

Any whole number can be written as a fraction by putting it over a denominator of 1.

$$4 = \frac{4}{1}, \qquad 12 = \frac{12}{1}, \qquad 39 = \frac{39}{1}$$





EXERCISE 3

1. Change the following to mixed numbers.

(a) $\frac{19}{4}$ =

(b) $\frac{23}{5}$ =

(b) $\frac{23}{3}$ =

(d) $\frac{46}{7}$ =

2. Change the following to improper fractions.

(a) $9\frac{2}{7}$ =

(b) $6\frac{4}{9}$ =

(c) $3\frac{1}{2}$ =

(d) $4\frac{3}{5}$ =

(e) $5\frac{1}{3}$ =

(f) 8 =

EQUIVALENT FRACTIONS

Reference Study page 6, column two of *Holt Mathematics 3.*

Rule

> To form a fraction equivalent to a given fraction, multiply (or divide) both the numerator and denominator by the same number.

Example Write a fraction equivalent to $\frac{5}{7}$ using a denominator of 112.

$$\frac{5}{7} = \frac{?}{112}$$

$$\frac{5}{7} \overset{\times 16}{=} \frac{80}{112}$$

Since the denominator is multiplied by 16, multiply the numerator by 16.

$\frac{5}{7}$ is equivalent to $\frac{80}{112}$.

EXERCISE 4

1. Form equivalent fractions by completing the following.

(a) $\frac{5}{11} = \frac{65}{\quad}$ (b) $\frac{16}{60} = \frac{\quad}{15}$

(c) $\frac{\quad}{28} = \frac{9}{7}$ (d) $\frac{\quad}{6} = \frac{12}{72}$

2. For each of the following write an equivalent fraction using a denominator of 189.

(a) $\frac{4}{63} =$ (b) $\frac{7}{9} =$

(c) $\frac{13}{21} =$ (d) $\frac{31}{27} =$

PRIME FACTORIZATION

Prime numbers have two factors, one and the number itself. The set of prime numbers is {2, 3, 5, 7, 11, 13, 17... }.

Finding Prime Factors

To find the prime factors of a number:

1. Divide the original number and each of the resulting quotients by the smallest prime number possible.
2. Continue this process until a quotient of 1 is obtained.
3. The divisors are the prime factors.

Example

Write 168 as a product of its prime factors.

$$\begin{array}{r|l} 2 & 168 \\ 2 & 84 \\ 2 & 42 \\ 3 & 21 \\ 7 & 7 \\ & 1 \end{array}$$

Notice that the division is done in a downhill format. 168 is equal to the product of the divisors.

$168 = 2 \times 2 \times 2 \times 3 \times 7$

The above method requires you to check if any of the prime numbers divide evenly into the given number. Here are some easy ways for checking if the first three primes divide evenly into the given numbers.

Divisibility by 2, 3 and 5

A number is divisible by two if it ends in the even numbers 0, 2, 4, 6, or 8.

A number is divisible by three, if the sum of its digits is divisible by 3. For example, 2214 is divisible by 3, because 2 + 2 + 1 + 4 = 9 and 9 is divisible by 3.

A number is divisible by 5 if it ends in either 0 or 5.

SELF-CORRECTING EXERCISE A

Write the following as products of their prime factors.

1. 98

2. 144

Check your answers with those on page 21 of this lesson. Correct any errors in your work.

FINDING THE GCF

The greatest common factor (GCF) is the largest number that is a factor of each of a given set of numbers.

One method of finding the GCF is to write each number as a product of its prime factors. Then the common factors found in each number are selected. The product of these factors is the GCF.

Example Find the GCF of 36 and 96.

$36 = 2 \times 2 \times 3 \times 3$

$96 = 2 \times 2 \times 2 \times 2 \times 2 \times 3$

2 | 36
2 | 18
3 | 9
3 | 3
1

2 | 96
2 | 48
2 | 24
2 | 12
2 | 6
3 | 3
1

The common factors have been circled. The GCF is the product of these common factors.

$GCF = 2 \times 2 \times 3$

$= 12$

Notice that 2, 2, and 3 appear in both 36 and 96.

EXERCISE 5 Find the GCF of the following sets of numbers.

1. 28, 42

2. 144, 150

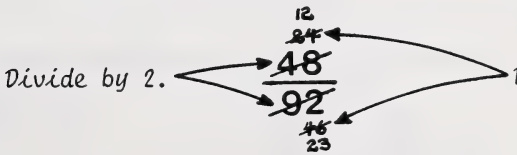
3. 26, 38, 44

4. 36, 52, 60

REDUCING FRACTIONS

A fraction can be reduced by dividing both the numerator and denominator by common factors. When the numerator and denominator have no factors in common, except 1, the fraction is said to be in lowest terms.

Example 1 Reduce $\frac{48}{92}$ to lowest terms.



Since 12 and 23 have no other factors in common, $\frac{12}{23}$ is in lowest terms.

Sometimes finding the common factors of a numerator and denominator can be quite difficult. Another approach is to write both the numerator and denominator as products of their prime factors. Then, the common factors are easily seen and you simply cancel them out.

Example 2 Write $\frac{96}{120}$ in lowest terms.

$$\frac{96}{120} = \frac{2 \times 2 \times 2 \times 2 \times 2 \times 3}{2 \times 2 \times 2 \times 3 \times 5}$$

Write the numerator and denominator as products of their prime factors.

$$= \frac{\overset{1}{\cancel{2}} \times \overset{1}{\cancel{2}} \times \overset{1}{\cancel{2}} \times 2 \times 2 \times \overset{1}{\cancel{3}}}{\underset{1}{\cancel{2}} \times \underset{1}{\cancel{2}} \times \underset{1}{\cancel{2}} \times \underset{1}{\cancel{3}} \times 5}$$

Cancel out the common factors.

$$= \frac{4}{5}$$

The product of the remaining factors is the fraction in lowest terms.

EXERCISE 6 1. Reduce to lowest terms.

(a) $\frac{57}{76}$ (b) $\frac{32}{128}$

(c) $\frac{90}{165}$ (d) $\frac{63}{147}$

2. Write the following as mixed numbers in lowest terms.

(a) $\frac{20}{6} = 3\frac{\overset{1}{\cancel{2}}}{\underset{3}{\cancel{6}}} = 3\frac{1}{3}$ (b) $\frac{124}{9} =$

(c) $\frac{51}{34} =$ (d) $\frac{63}{14} =$

FINDING THE LCM

The lowest common multiple (L.C.M.) is the smallest number that is a multiple of each of the given sets of numbers.

Three methods may be used to find the L.C.M.

In the following two examples the L.C.M. is found in 3 different ways. Study these examples and choose the method you prefer.

Example I: Find the L.C.M. of 42 and 60.

Method I: List the multiples of 42 and 60.

Multiples of 42: 42, 84, 126, 168, 160, 252, 294, 336, 378, 420...

Multiples of 60: 60, 120, 180, 240, 300, 360, 420,...

L.C.M. = 420

Method II: Write 42 and 60 as a product of prime factors.

$42 = 2 \times 3 \times 7$

$60 = 2 \times 2 \times 3 \times 5$

$$\begin{array}{r|l} 2 & 42 \\ 3 & 21 \\ & 7 \end{array} \qquad \begin{array}{r|l} 2 & 60 \\ 2 & 30 \\ 3 & 15 \\ & 5 \end{array}$$

The common factors have been circled. The L.C.M. is the product of these common factors and the remaining prime factors.

$$\text{L.C.M.} = 2 \times 3 \times 2 \times 5 \times 7$$
$$= 420$$

Method III: We can simplify Method II in the following way.

$$\begin{array}{r|l} (2) & 42,\ 60 \\ (3) & 21,\ 30 \\ & 7,\ 10 \end{array}$$

The common factors have been circled. The L.C.M. is the product of these common factors and the remaining factors.

$$\text{L.C.M.} = 2 \times 3 \times 7 \times 10 = 420$$

Example II: Find the L.C.M. of 16, 28, 24

Method I: List the multiples of 16, 18, 24.

Multiples of 16: 16, 32, 48, 64, 80, 96, 112, 128, 144,...

Multiples of 18: 18, 36, 54, 72, 90, 108, 126, 144,...

Multiples of 24: 24, 48, 72, 96, 120, 144,...

L.C.M. = 144

Method II: WRite 16, 18, 24 as a product of prime factors.

$16 = 2 \times 2 \times 2 \times 2$
$18 = 2 \times 3 \times 3$
$24 = 2 \times 2 \times 2 \times 3$

$2 \mid 16$	$2 \mid 18$	$2 \mid 24$
$2 \mid 8$	$3 \mid 9$	$2 \mid 12$
$2 \mid 4$	3	$2 \mid 6$
2		3

The common factors have been circled. The L.C.M. is the product of these common factors and the remaining prime factors.

L.C.M. = 2 x 2 x 2 x 3 x 2 x 3
= 144

Method III: We can simplify Method II in the following way.

$2 \mid 16, 18, 24$
$2 \mid 8, 9, 12$ ← because 2 is not a factor of 9, 9 is repeated.
$2 \mid 4, 9, 6$
$3 \mid 2, 9, 3$ ← because 3 is not a factor of 2, 2 is repeated.
2, 3, 1

The common factors have been circled. The L.C.M. is the product of these common factors and the remaining factors.

L.C.M. = 2 x 2 x2 x 3 x 2 x 3 x 1 = 144.

SELF-CORRECTING EXERCISE B

Find the least common multiple of the following sets of numbers.

1. 28, 40

2. 15, 32

Check your answers with those on page 21 of this lesson. Correct any errors in your work.

ADDING AND SUBTRACTING FRACTIONS

Reference

Study Examples 1, 2 and 3 on page 16 of *Holt Mathematics 3*.

Fractions cannot be added or subtracted unless they have the same denominator.

Method

To add or subtract fractions that have different denominators:

1. Calculate the LCM of the denominators.
2. *Use the LCM to write equivalent fractions for each of the given fractions.
3. Add or subtract as indicated.

*Notice that your textbook uses a different approach to doing step 2. Either method is acceptable.

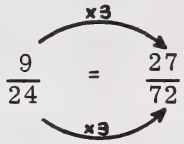
Example 1 Add $\frac{9}{24}$ and $\frac{14}{36}$.

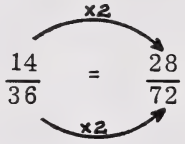
First, find the LCM of the denominators.

$24 = 2 \times 2 \times 2 \times 3$ LCM $= 2 \times 2 \times 2 \times 3 \times 3$

$36 = 2 \times 2 \times 3 \times 3$ $= 72$

Second, write equivalent fractions using 72 as the denominator.





Third, add the fractions.

$$\frac{9}{24} + \frac{14}{36}$$
$$= \frac{27}{72} + \frac{28}{72}$$
$$= \frac{55}{72}$$

SELF-CORRECTING EXERCISE C

1. (a) $\frac{1}{5}+\frac{3}{8}$ (b) $\frac{2}{5}+\frac{1}{4}$

2. (a) $\frac{5}{8}-\frac{5}{12}$ (b) $\frac{5}{6}-\frac{1}{4}$

Check your answers with those on page 24 of this lesson.
Correct any errors in your work.

EXERCISE 7 Compute as indicated. Write your answers in lowest terms.

1. $\frac{5}{18} + \frac{3}{20}$

2. $\frac{13}{12} - \frac{5}{16}$

3. $\frac{20}{34} + \frac{11}{51}$

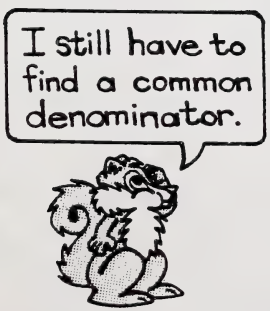
4. $\frac{84}{160} - \frac{5}{32}$

ADDING MIXED NUMBERS

Reference Study Example 4 on page 16 of *Holt Mathematics 3.*

Recall that a mixed number is made up of a whole number plus a fraction. To add mixed numbers, first change to improper fractions.

Example Add $2\frac{5}{6}$ and $1\frac{3}{4}$



$$2\frac{5}{6} + 1\frac{3}{4} = \frac{17}{6} + \frac{7}{4}$$

$$= \frac{34}{12} + \frac{21}{12}$$

$$= \frac{55}{12}$$

$$= 4\frac{7}{12}$$

SELF-CORRECTING EXERCISE D

Compute. Your final answer must be in lowest terms.

1. (a) $2\frac{1}{5}+4\frac{2}{3}$ (b) $6\frac{5}{8}+7\frac{11}{12}$

Check your answers with those on page 24 of this lesson.
Correct any errors in your work.

EXERCISE 8 Add the following. Write your answers in lowest terms.

1. $2\frac{5}{8} + 9\frac{2}{7} = \frac{21}{8} + \frac{65}{7}$

$= \frac{147}{56} + \frac{520}{56}$

$= \frac{667}{56}$

$= 11\frac{53}{56}$

2. $1\frac{3}{4} + 4\frac{9}{10}$

3. $5\frac{1}{6} + 2\frac{11}{15}$

4. $1\frac{9}{14} + 1\frac{2}{7}$

SUBTRACTING MIXED NUMBERS

To subtract mixed numbers, you use the same approach used in adding mixed numbers.

Example 1 Calculate $1\frac{5}{6} - \frac{2}{8}$.

$$1\frac{5}{6} - \frac{2}{8} = \frac{11}{6} - \frac{2}{8}$$

$$= \frac{44}{24} - \frac{6}{24}$$

$$= \frac{38}{24}$$

Express in lowest terms.

$$= \frac{19}{12}$$

OR $1\frac{7}{12}$ Write as a mixed number if you wish.

SELF-CORRECTING EXERCISE E

Compute. Your final answer must be in lowest terms.

1. (a) $2\frac{3}{4} - 1\frac{5}{6}$ (b) $5\frac{3}{8} - 3\frac{3}{4}$

(c) $9\frac{3}{4} - 2\frac{1}{3}$ (d) $5\frac{3}{10} - 1\frac{2}{3}$

Check your answers with those on page 24 of this lesson. Correct any errors in your work.

EXERCISE 9

Subtract the following. Write your answers in lowest terms.

1. $2\frac{1}{2} - 1\frac{3}{5} = \frac{5}{2} - \frac{8}{5}$

$$= \frac{25}{10} - \frac{16}{10}$$

$$= \frac{9}{10}$$

2. $10\frac{3}{8} - 6\frac{1}{2} =$

3. $8\frac{4}{9} - 6\frac{2}{3} =$

4. $3\frac{5}{12} - 1\frac{7}{8} =$

RATIONAL NUMBERS

Reference

Study page 66 of **Holt Mathematics 3.**

Negative Rationals

Example 2 on page 66 of **Holt Mathematics 3** shows.

$$\frac{-1}{2} = \frac{1}{-2} = \frac{1}{-2}$$

Thus, you can see that the negative sign can be placed with the numerator or, with the denominator or, in front of the rational number.

However, $\frac{-1}{-2}$ is not a negative number. Recall that a negative divided by a negative is positive. Thus, $\frac{-1}{-2}$ equals $+\frac{1}{2}$ or $\frac{+1}{+2}$.

Sometimes you will be working with rationals in the mixed number form. You will want to change the mixed number to an improper fraction first.

$$\begin{aligned} -3\frac{1}{2} &= -3 + \left(-\frac{1}{2}\right) \\ &= \frac{-6}{2} + \left(-\frac{1}{2}\right) \\ &= -\frac{7}{2} \\ &= -\frac{7}{2} \end{aligned}$$

Shortcut

$-3\frac{1}{2} = -\left(3\frac{1}{2}\right)$ so the short cut method on page 4 may be used.

$-3\frac{1}{2} = -\left(3\frac{1}{2}\right) = -\left(\frac{7}{2}\right) = -\frac{7}{2}$

ADDITION AND SUBTRACTION OF RATIONAL FRACTIONS

Reference: Study Examples 2 and 3 on page 68 of **Holt Mathematics 3.**

The rules for adding or subtracting rational numbers are the same as the rules used for fractions. However, you must also use the rules for adding and subtracting integers.

Example 1 Add $-\frac{9}{14}$ and $-\frac{1}{5}$.

$$\frac{-9}{14} + \frac{-1}{5} = \frac{-45}{70} + \frac{-14}{70}$$

Be careful to put in the proper signs.

$$= \frac{-45 + -14}{70}$$

$$= \frac{-59}{70}$$

To add the numerators, use the rules for adding integers.

Example 2 Add $-1\frac{3}{8}$ and $\frac{5}{6}$.

$$-1\frac{3}{8} + \frac{5}{6} = \frac{-11}{8} + \frac{5}{6}$$

Change to an improper fraction.

$$= \frac{-33}{24} + \frac{20}{24}$$

$$= \frac{-13}{24}$$

Example 3 Calculate $-\frac{3}{4} - \frac{1}{2}$.

$$-\frac{3}{4} - \frac{1}{2} = \frac{-3}{4} + \left(\frac{-1}{2}\right)$$

Change to addition first.

$$= \frac{-3}{4} + \frac{-1}{2}$$

$$= \frac{-3}{4} + \frac{-2}{4}$$

$$= \frac{-5}{4}$$

Example 4 Calculate $\frac{-5}{6} - \left(\frac{-2}{3}\right)$

$$\frac{-5}{6} - \left(\frac{-2}{3}\right) = \frac{-5}{6} + \left(\frac{+2}{3}\right)$$ ← Change to addition first.

$$= \frac{-5}{6} + \frac{4}{6}$$

$$= \frac{-1}{6}$$

Example 5 Calculate $+3\ \frac{1}{2} - \left(-1\ \frac{1}{4}\right)$

$$+3\ \frac{1}{2} - \left(-1\ \frac{1}{4}\right) = +3\ \frac{1}{2} + \left(+1\ \frac{1}{4}\right)$$ ← Change to addition.

$$= \frac{7}{2} + \frac{5}{4}$$ ← Change to improper fraction.

$$= \frac{14}{4} + \frac{5}{4}$$

$$= \frac{19}{4}$$

SELF-CORRECTING EXERCISE F

Evaluate.

1. (a) $\frac{2}{5}+\left(\frac{-7}{10}\right)$ (b) $\left(\frac{-5}{6}\right)+\frac{1}{3}$

(c) $\left(\frac{-3}{8}\right)+\frac{-1}{4}$ (d) $\frac{-3}{10}+\frac{5}{10}$

(e) $-1\frac{1}{2}-1\frac{1}{4}$ (f) $\frac{-1}{2}+2\frac{1}{4}$

Check your answers with those on page 25 of this lesson.
Correct any errors in your work.

EXERCISE 10 Compute the following.

1. $-\frac{1}{5} + \frac{1}{2}$

2. $\frac{2}{3} - \frac{4}{5}$

3. $\frac{3}{5} - \left(\frac{-1}{10}\right)$

4. $\frac{-3}{8} + 2\frac{1}{4}$

5. $\frac{-9}{10} + \left(-2\frac{2}{5}\right)$

6. $2 - 4\frac{3}{4}$

KEY TO SELF-CORRECTING EXERCISES IN LESSON 3

SELF-CORRECTING EXERCISE A, page 8.

1.
$$\begin{array}{r|l} 2 & 98 \\ 7 & 49 \\ 7 & 7 \\ & 1 \end{array}$$

2.
$$\begin{array}{r|l} 2 & 144 \\ 2 & 72 \\ 2 & 36 \\ 2 & 18 \\ 3 & 9 \\ 3 & 3 \\ & 1 \end{array}$$

SELF-CORRECTING EXERCISE B, page 11.

1. $28 = 2 \times 2 \times \underline{7}$

   $40 = \underline{2 \times 2 \times 2} \times \underline{5}$

   $\text{LCM} = 2 \times 2 \times 2 \times 5 \times 7$
   $= 280$

2. $15 = \underline{3} \times \underline{5}$

   $32 = \underline{2 \times 2 \times 2 \times 2 \times 2}$

   $\text{LCM} = 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 5$
   $= 480$

Notice that the largest group of each factor has been underlined.

SELF-CORRECTING EXERCISE C, page 14.

1. (a) $\frac{1}{5}+\frac{3}{8}=\frac{8}{40}+\frac{15}{40}$

$=\frac{23}{40}$

(b) $\frac{2}{5}+\frac{1}{4}=\frac{8}{20}+\frac{5}{20}$

$=\frac{13}{20}$

2. (a) $\frac{5}{8}-\frac{5}{12}=\frac{15}{24}-\frac{10}{24}$

$=\frac{5}{24}$

(b) $\frac{5}{6}-\frac{1}{4}=\frac{10}{12}-\frac{3}{12}$

$=\frac{7}{12}$

SELF-CORRECTING EXERCISE D, page 16.

1. (a) $2\frac{1}{5}+4\frac{2}{3}=2\frac{3}{15}+4\frac{10}{15}$

$=6\frac{13}{15}$

(b) $6\frac{5}{8}+7\frac{11}{12}=6\frac{15}{24}+7\frac{22}{24}$

$=13\frac{37}{24}$

$=14\frac{13}{24}$

SELF-CORRECTING EXERCISE E, page 17.

1. (a) $2\frac{3}{4}-1\frac{5}{6}=\frac{11}{4}-\frac{11}{6}$

$=\frac{33}{12}-\frac{22}{12}$

$=\frac{11}{12}$

(b) $5\frac{3}{8}-3\frac{3}{4}=\frac{43}{8}-\frac{15}{4}$

$=\frac{43}{8}-\frac{30}{8}$

$=\frac{13}{8}$

$=1\frac{5}{8}$

(c) $9\frac{3}{4}-2\frac{1}{3}=\frac{39}{4}-\frac{7}{3}$

$=\frac{117}{12}-\frac{28}{12}$

$=\frac{89}{12}$

$=7\frac{5}{12}$

(d) $5\frac{3}{10}-1\frac{2}{3}=\frac{53}{10}-\frac{5}{3}$

$=\frac{159}{30}-\frac{50}{30}$

$=\frac{109}{30}$

$=3\frac{19}{30}$

SELF-CORRECTING EXERCISE F, page 21.

1. (a) $\frac{2}{5}+\frac{-7}{10}=\frac{4}{10}+\frac{-7}{10}$

$=\frac{-3}{10}$

(b) $\frac{-5}{6}+\frac{1}{3}=\frac{-5}{6}+\frac{2}{6}$

$=\frac{-3}{6}$

$=\frac{-1}{2}$

(c) $\frac{-3}{8}+\frac{-1}{4}=\frac{-3}{8}+\frac{-2}{8}$

$=\frac{-5}{8}$

(d) $\frac{-3}{10}+\frac{5}{10}=\frac{2}{10}$

$=\frac{1}{5}$

(e) $-1\frac{1}{2}-1\frac{1}{4}=\frac{-3}{2}+\frac{-5}{4}$

$=\frac{-6}{4}+\frac{-5}{4}$

$=\frac{-11}{4}$

$=-2\frac{3}{4}$

(f) $\frac{-1}{2}+2\frac{1}{4}=\frac{-1}{2}+\frac{9}{4}$

$=\frac{-2}{4}+\frac{9}{4}$

$=\frac{7}{4}$

$=1\frac{3}{4}$

END OF LESSON 3

LESSON RECORD FORM

0902 Math 9
Revised 11/90

FOR STUDENT USE ONLY

Date Lesson Submitted

_____

Time Spent on Lesson

_____

(If label is missing or incorrect)

File Number

_____

Lesson Number _____

Student's Questions and Comments

Apply Lesson Label Here

Name

Address

Postal Code

Please verify that preprinted label is for correct course and lesson.

FOR A.D.L.C. USE ONLY

Assigned Teacher: _____

Lesson Grading: _____

Additional Grading E/R/P Code: _____

Mark: _____

Graded by: _____

Assignment Code: _____

Date Lesson Received:

_____

Lesson Recorded _____

Teacher's Comments:

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Correspondence Teacher

St. Serv. 14-91

ALBERTA DISTANCE LEARNING CENTRE

MAILING INSTRUCTIONS FOR CORRESPONDENCE LESSONS

1. BEFORE MAILING YOUR LESSONS, PLEASE SEE THAT:

(1) All pages are numbered and in order, and no paper clips or staples are used.
(2) All exercises are completed. If not, explain why.
(3) Your work has been re-read to ensure accuracy in spelling and lesson details.
(4) The Lesson Record Form is filled out and the correct lesson label is attached.
(5) This mailing sheet is placed on the lesson.

2. POSTAGE REGULATIONS

Do not enclose letters with lessons.

Send all letters in a separate envelope.

3. POSTAGE RATES

First Class

Take your lesson to the Post Office and have it weighed. Attach sufficient postage and a green first-class sticker to the front of the envelope, and seal the envelope. Correspondence lessons will travel faster if first-class postage is used.

Try to mail each lesson as soon as it has been completed.

When you register for correspondence courses, you are expected to send lessons for correction regularly. Avoid sending more than two or three lessons in one subject at the same time.

St. Serv. 9-91

OPERATIONS WITH FRACTIONS, DECIMALS and RATIONAL NUMBERS

In this lesson you will learn about:

- multiplying and dividing fractions, mixed numbers and rational numbers
- writing decimal numbers
- rounding off decimals
- adding, subtracting, multiplying and dividing decimals

KEEPING SKILLFUL

EXERCISE 1

1. Perform the following operations.

   (a) $-6 \times 3 =$ (b) $(-24)(-8) =$

   (c) $-152 \div 4 =$ (d) $-381 \div -3 =$

2. Write the following as mixed numbers in lowest terms.

   (a) $\frac{38}{6} =$ (b) $\frac{385}{2} =$

3. Write the following as improper fractions.

   (a) $9\frac{3}{7} =$ (b) $10\frac{1}{5} =$

4. Reduce these fractions to lowest terms.

   (a) $\frac{15}{35} =$ (b) $\frac{45}{96} =$

5. Round off the following to the nearer hundred.

   (a) 209 = _____

   (b) 3645 = _____

MULTIPLYING FRACTIONS AND MIXED NUMBERS

Rule

> To multiply fractions, multiply their numerators, then multiply their denominators.

To avoid having to reduce the product to lowest terms, it is usually easier to simplify before you multiply. When simplifying, you can reduce any numerator with any denominator by dividing by a common factor.

Example 1 Find the product of $\frac{14}{25}$ and $\frac{10}{12}$.

$$\frac{14}{25} \times \frac{10}{12} = \frac{\cancel{14}^{7} \times \cancel{10}^{2}}{\cancel{25}_{5} \times \cancel{12}_{6}}$$

$$= \frac{7 \times 1}{5 \times 3}$$

$$= \frac{7}{15}$$

To simplify:

- *five divides into 10 and 25,*
- *two divides into 14 and 12,*
- *and two divides into the remaining 2 and 6.*

Mixed Numbers

To multiply mixed numbers, you must change each mixed number to an improper fraction. Lesson 3, page 4, explains the method for doing this.

Also remember that whole numbers can be written as improper fractions by putting them over a denominator of 1. For example, 6 equals $\frac{6}{1}$ as an improper fraction.

Reference

Study page 15 of *Holt Mathematics 3.*

Example 2 Find the product of $1\frac{7}{18}$ and $3\frac{7}{11}$.

$$1\frac{7}{18} \times 3\frac{7}{11} = \frac{25}{18} \times \frac{40}{11}$$

The mixed numbers are changed to improper fractions.

$$= \frac{25 \times \overset{20}{\cancel{40}}}{\underset{9}{\cancel{18}} \times 11}$$

When simplifying, the 25 and 20 can not be reduced by 5 because both numbers are in the numerator.

$$= \frac{500}{99}$$

$$= 5\frac{5}{99}$$

SELF-CORRECTING EXERCISE A

Find the following products.

1. (a) $7 \times \frac{1}{3}$ (b) $\frac{5}{6} \times 8$

(c) $\frac{1}{4} \times \frac{2}{3}$ (d) $\frac{1}{6} \times \frac{6}{7}$

(e) $1\frac{1}{4} \times 8$ (f) $1\frac{2}{3} \times 2\frac{1}{4}$

Check your answers with those on page 21 of this lesson.
Correct any errors in your work.

EXERCISE 2 Find the following products. Put the answers in lowest terms.

1. $\frac{9}{15} \times \frac{8}{30}$

2. $\frac{21}{35} \times \frac{10}{1}$

3. $\frac{3}{5} \times 6$

4. $40 \times 2\frac{1}{5}$

5. $3\frac{3}{7} \times \frac{8}{3}$

6. $5\frac{2}{3} \times 1\frac{3}{34}$

RECIPROCALS

Reference — Study page 19, column 1, of *Holt Mathematics 3.*

If $a \times b = 1$, then a and b are reciprocals of each other. For example, $\frac{8}{7}$ and $\frac{7}{8}$ are reciprocals of each other because $\frac{8}{7} \times \frac{7}{8} = 1$.

Note

> The reciprocal of a fraction can be obtained by interchanging the numerator and denominator.

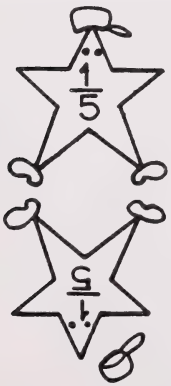
To find the reciprocals of whole numbers or mixed numbers you must first write them as improper fractions. For example, reciprocal of 2, or $\frac{2}{1}$, is $\frac{1}{2}$. Also, the reciprocal of $3\frac{2}{5}$, or $\frac{17}{5}$, is $\frac{5}{17}$.

DIVIDING FRACTIONS AND MIXED NUMBERS

Rule

> To divide by a fraction, multiply by its reciprocal.

Example 1 — Find the quotient of $\frac{9}{24}$ and $\frac{1}{5}$.



$$\frac{9}{24} \div \frac{1}{5} = \frac{9}{24} \times \frac{5}{1}$$ *Change dividing to multiplying by the reciprocal of the divisor.*

$$= \frac{\overset{3}{\cancel{9}} \times 5}{\underset{8}{\cancel{24}} \times 1}$$ *Simplify.*

$$= \frac{15}{8}$$

$$= 1\frac{7}{8}$$

Mixed Numbers

To divide by mixed numbers (or whole numbers), first change them to improper fractions. Then, change the divisor to its reciprocal and multiply.

Example 2 Find the quotient of $9\frac{3}{8}$ and $4\frac{1}{6}$.

$$9\frac{3}{8} \div 4\frac{1}{6} = \frac{75}{8} \div \frac{25}{6}$$

Change the mixed numbers to improper fractions.

$$= \frac{\cancel{75}^{\,3}}{\cancel{8}_{\,4}} \times \frac{\cancel{6}^{\,3}}{\cancel{25}_{\,1}}$$

Change the divisor to its reciprocal and multiply.

$$= \frac{9}{4}$$

$$= 2\frac{1}{4}$$

EXERCISE 3 Find the quotient for each of the following. Write your answers in lowest terms.

1. $\frac{9}{8} \div \frac{1}{2}$

2. $\frac{5}{16} \div 4$

3. $3\frac{1}{8} \div \frac{5}{7}$

4. $3\frac{3}{5} \div 7\frac{5}{7}$

MULTIPLYING AND DIVIDING RATIONAL NUMBERS

Reference Study pages 70 and 71 of *Holt Mathematics 3.*

To multiply or divide rational numbers use the same rules that were used for fractions. However, you must also apply the rules for integers. These rules are summarized in a small box on page 71 of *Holt Mathematics 3.*

From Examples 3 and 4 on pages 70 and 71 of your textbook, you should note that the reciprocal of a negative number is negative and the reciprocal of a positive number is positive. This sign does not change when you invert the number.

SELF-CORRECTING EXERCISE B

Find the following products. Put the answers in their simplest form.

1. (a) $1\frac{1}{5} \times -2\frac{1}{2}$ (b) $\frac{2}{3} \times \frac{-5}{4}$

2. (a) $1\frac{1}{2} \div -2$ (b) $\frac{-3}{8} \div -3\frac{1}{4}$

Check your answers with those on page 21 of this lesson. Correct any errors in your work.

EXERCISE 4 Do the following operations. Write your answers in lowest terms.

1. $-3\frac{2}{5} \times 4$

2. $-2\frac{1}{2} \times -4\frac{3}{4}$

3. $20\frac{2}{3} \times -\frac{24}{31}$

4. $-15 \div \frac{3}{4}$

5. $-6\frac{1}{5} \div -4\frac{1}{2}$

6. $-1\frac{3}{7} \div 11\frac{1}{4}$

DECIMALS

Reference Read page 21 of *Holt Mathematics 3.*

Place Values The place values of the decimal number 35 428.209 16 are given below

3	5	4	2	8	.	2	0	9	1	6
ten-thousands	thousands	hundreds	tens	ones		tenths	hundredths	thousandths	ten-thousandths	hundred-thousandths

Notice that the place values that come after the decimal point and are compound words are hyphenated.

Also, each place value that comes after the decimal point ends in "ths".

SELF-CORRECTING EXERCISE C

Give the place value of each underlined digit.

1. (a) 23.<u>9</u>5 (b) 0.38<u>5</u>

(c) 29.010<u>2</u> (d) 0.03<u>4</u>2

(e) <u>7</u>00 000 (f) 321.3612<u>8</u>

Check your answers with those on page 22 of this lesson. Correct any errors in your work.

WRITING DECIMAL NUMBERS

Whole Number Part

In our numeration system, whole numbers are grouped in threes, going from right to left. Each group of three digits is called a period.

When you write a whole number in digits, spaces are used to separate each period. Similarly, when you write a whole number in words a comma is used to indicate the end of each period.

Example

2 914 465

Notice the spaces.

two million, nine hundred fourteen thousand, four hundred sixty-five

Decimal Part

When you write a decimal number in digits, group the digits after the decimal point in threes, going from left to right. Use spaces to separate these groups of digits.

Example

0.483 6 *Notice the space.*

Unlike the space in whole numbers, this space does not indicate where to put a comma when you write the decimal part in words.

> When writing a decimal number in words the decimal part is treated as a whole number, which is followed by the place value of the last digit.

Consider the decimal number 0.483 6. You read it like the whole number 4 836 followed by the word ten-thousandths. Thus, 0.483 6 is read as: four thousand, eight hundred thirty-six ten-thousandths.

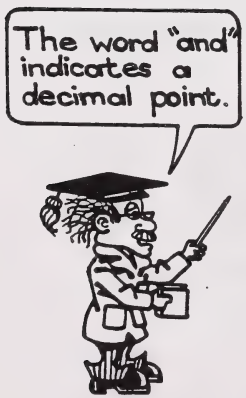
In a decimal number, the whole number part and the decimal part are read separately, but are connected by the word *and*.

Never use the word *and* unless you want to indicate the decimal point.

Examples		
	2065.19	two thousand, sixty-five and nineteen hundredths
	6.023 81	six and two thousand, three hundred eighty-one hundred-thousandths
	0.300	three hundred thousandths
	0.000 03	three hundred-thousandths

Look at the last two examples above. The words are indentical, except for the hyphen. Thus, it is important to *hyphenate place values that are compound words and come after the decimal point*. Also notice, when the whole number is zero, there is no need to write "zero and ...".

EXERCISE 5 1. Write these numbers using digits. Use proper spacing.



_____ (a) eleven and thirty-four thousandths

_____ (b) five thousand, four hundred two ten-thousandths

_____ (c) five thousand, four hundred and two ten-thousandths

_____ (d) five thousand and four hundred two ten-thousandths

_____ (e) thirty-three and twenty-four thousandths

2. Write the following numbers in words.

(a) 94.006 _____

_____

_____

(b) 8.239 5 _____

_____

_____

(c) 203.100 54 _____

_____

ROUNDING OFF DECIMALS

Rule

To round off a number to a given place value, look at the digit immediately to the right of that place value. Then:

- if the number to the right is 5 or larger, round up the digit in the given place value by one.

- if the number to the right is less than 5, leave the digit in the given place value as it is.

Examples

16.207 = 16.2 (to the nearer tenth)

The zero does not cause the 2 to round up.

4.851 = 4.9 (to the nearer tenth)

The five rounds the 8 up to 9.

24.096 = 24.10 (to the near hundredth)

The 6 rounds the 9 up to 10.

EXERCISE 6 Round off the following to the indicated place values.

1. 98.439 = _____ (to the nearer tenth)
2. 4.605 2 = _____ (to the nearer tenth)
3. 173.98 = _____ (to the nearer tenth)
4. 2.064 5 = _____ (to the nearer hundredth)
5. 0.100 9 = _____ (to the nearer hundredth)
6. 36.123 4 = _____ (to the nearer thousandth)
7. 5.048 62 = _____ (to the nearer ten-thousandth)
8. 44.506 = _____ (to the nearer whole number)

ADDING AND SUBTRACTING DECIMALS

When adding or subtracting decimals, it is important to line up the decimal points.

Remember that the decimal comes at the end of a whole number. Zeros as placeholders can be put in after the decimal point.

SELF-CORRECTING EXERCISE D

Compute:

1. (a) 2.6 + 3.12 + 52 (b) 23.014 - 1.95

Check your answers with those on page 22 of this lesson.
Correct any errors in your work.

MULTIPLYING DECIMALS

To multiply decimal numbers, multiply as you normally would. Do not line up the decimal points. Then follow the rule below.

Rule

> The sum of the number of decimal places in the factors equals the number of decimal places in the product.

Example

$$
\begin{array}{rl}
3.04 & \leftarrow \text{2 decimal places plus} \\
\times 5.3 & \leftarrow \text{1 decimal place.} \\
\hline
912 & \\
1520 & \\
\hline
16.112 & \leftarrow \text{3 decimal places in the product.}
\end{array}
$$

SELF-CORRECTING EXERCISE E

Calculate:

1. (a) 3.12×1.8 (b) 3.012×0.41

Check your answers with those on page 22 of this lesson.
Correct any errors in your work.

EXERCISE 7 Perform the following operations.

1. $21 + 3.4 + 8.063$

2. $0.039 + 2 + 11.6$

3. $36.2 - 1.45$

4. $42 - 6.084$

5. 0.26×2.45

6. 30.6×0.984

DIVIDING DECIMALS

The parts of a division question are shown below.

$$\begin{array}{r} 2.4 \leftarrow \textit{quotient} \\ \textit{divisor} \rightarrow 3\overline{)\,7.2} \leftarrow \textit{dividend} \end{array} \quad \text{OR} \quad \textit{dividend} \;\; \textit{divisor} \;\; 7.2 \div 3 = 2.4 \leftarrow \textit{quotient}$$

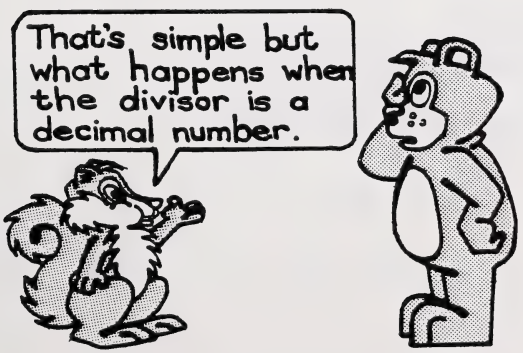
Reference — Study Examples 1, 2, 3 and 4 on page 27 of *Holt Mathematics 3.*

When you divide a whole number into a decimal number, simply place a decimal point in the quotient directly above its position in the dividend.

Example 1 — Divide 4.2 by 8.

$$\begin{array}{r} 0.525 \\ 8\overline{)\,4.200} \\ 4\;0\times\times \\ \hline 20 \\ 16 \\ \hline 40 \\ 40 \\ \hline 0 \end{array}$$

Extra zeros can be added after the decimal point.



If the divisor is a decimal number, it must be changed to a whole number. To do this, move the divisor's decimal point to the end of the number. Then move the decimal point in the dividend the same number of places as it was moved in the divisor.

Example 2 Divide 6.017 by 0.55 .

$$
\begin{array}{r}
10.94 \\
0.55\overset{\wedge}{)}\ 6.0170 \\
\underline{5\ 5} \\
51 \\
\underline{00} \\
517 \\
\underline{495} \\
220 \\
\underline{220} \\
0
\end{array}
$$

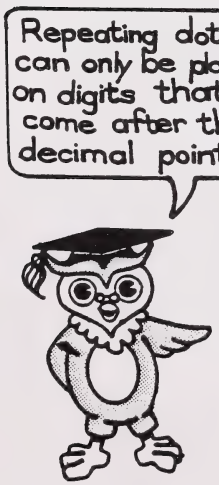
The symbol ^ (named the caret) shows that the decimal was moved 2 places in the divisor and dividend.

For each of the examples done so far the quotient terminated. This does not always happen. In some cases one or more digits will repeat over and over. To show this, repeating dots are used.

Examples

Divide 52 by 0.6 .

$$
\begin{array}{r}
86.6 \\
0.6\overset{\wedge}{)}\ 52.0\overset{\wedge}{0} \\
\underline{48} \\
40 \\
\underline{36} \\
40 \\
\underline{36} \\
4
\end{array}
$$



The 6 is repeating. The repeating dot is put above the first 6 that comes after the decimal point.

$52 \div 0.6 = 86.\dot{6}$

Divide 0.2 by 1.3 .

$$
\begin{array}{r}
0.153846\,1 \\
1.3\overset{\wedge}{)}\ 0.2000000 \\
\underline{13} \\
70 \\
\underline{65} \\
50 \\
\underline{39} \\
110 \\
\underline{104} \\
60 \\
\underline{52} \\
80 \\
\underline{78} \\
2
\end{array}
$$

The digits 153846 will repeat. A repeating dot is put above the first and last digit of this repeating block.

$0.2 \div 1.3 = 0.\dot{1}53846\dot{}$

Instead of calculating a quotient until the decimal repeats or terminates, you can round off a quotient to a given place value.

Reference

Study Example 5 on page 27 and Example 6 on page 28 of *Holt Mathematics 3.*

Notice that you must always compute your quotient to one place more than the place value you are asked to find. Then, round off your quotient as necessary.

SELF-CORRECTING EXERCISE F

Part A: For the following questions, indicate the new location of the decimal points by using the caret sign (∧). Question 1 has been done for you.

1. $0.089_{\wedge}\overline{)\ 2.300_{\wedge}}$

2. $0.9\overline{)11}$

3. $5.21\overline{)0.184}$

4. $0.002\overline{)14.8}$

Part B:

5. (a) $8.225 \div 0.05$ (b) $2.622 \div 5.7$

Check your answers with those on page 22 of this lesson.

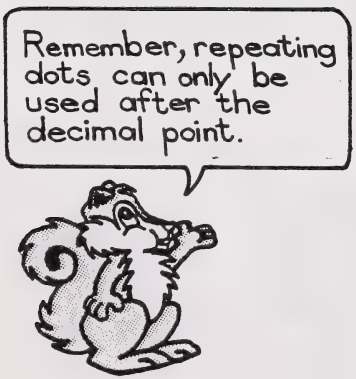
EXERCISE 8

1. Find the following quotients. Divide until the quotient terminates.

   (a) $0.045 \div 0.8 =$ _____ (b) $9.85 \div 1.6 =$ _____

2. Find the following quotients. Divide until the quotient repeats.

   (a) $23.6 \div 12 =$ _____ (b) $2.56 \div 3.3 =$ _____



3. Find the following quotients to the nearer tenth.

(a) $0.1 \div 0.64 =$ _____ (b) $2.4 \div 0009 =$ _____

KEY TO SELF-CORRECTING EXERCISES IN LESSON 4

SELF-CORRECTING EXERCISE A, page 4.

1. (a) $7 \times \frac{1}{3} = \frac{7}{3} = 2\frac{1}{3}$

(b) $\frac{5}{\cancel{6}_{3}} \times \frac{\cancel{8}^{4}}{1} = \frac{20}{3} = 6\frac{2}{3}$

(c) $\frac{1}{\cancel{4}_{2}} \times \frac{\cancel{2}^{1}}{3} = \frac{1}{6}$

(d) $\frac{1}{\cancel{6}_{1}} \times \frac{\cancel{6}^{1}}{7} = \frac{1}{7}$

(e) $\frac{5}{\cancel{4}_{1}} \times \frac{\cancel{8}^{2}}{1} = 10$

(f) $\frac{5}{\cancel{3}_{1}} \times \frac{\cancel{9}^{3}}{4} = \frac{15}{4} = 3\frac{3}{4}$

SELF-CORRECTING EXERCISE B, page 8.

1. (a) $1\frac{1}{5} \times -2\frac{1}{2} = \frac{\cancel{6}^{3}}{\cancel{5}_{1}} \times \frac{-\cancel{5}^{1}}{\cancel{2}_{1}} = -3$

(b) $\frac{\cancel{2}^{1}}{3} \times \frac{-5}{\cancel{4}_{2}} = \frac{-5}{6}$

2. (a) $1\frac{1}{2} \div -2 = \frac{3}{2} \times \frac{-1}{2} = \frac{-3}{4}$

(b) $\frac{-3}{8} \div -3\frac{1}{4} = \frac{-3}{8} \div \frac{-13}{4} = \frac{-3}{\cancel{8}_{2}} \times \frac{-\cancel{4}^{1}}{13} = \frac{3}{26}$

SELF-CORRECTING EXERCISE C, page 10.

1. (a) 23.$\underline{9}$5 tenths (b) 0.38$\underline{5}$ thousandths

   (c) 29.010$\underline{2}$ ten-thousandths (d) 0.034$\underline{2}$ thousandths

   (e) $\underline{7}$00 000 hundred thousands (f) 321.3612$\underline{8}$ hundred thousandths

SELF-CORRECTING EXERCISE D, page 14.

1. (a)
$$\begin{array}{r} 2.6 \\ 3.12 \\ 52. \\ \hline 57.72 \end{array}$$

(b)
$$\begin{array}{r} 23.014 \\ -1.95 \\ \hline 21.064 \end{array}$$

SELF-CORRECTING EXERCISE E, page 15.

1. (a)
$$\begin{array}{r} 3.12 \\ \times\ 1.8 \\ \hline 2496 \\ 312 \\ \hline 5.616 \end{array}$$

(b)
$$\begin{array}{r} 3.012 \\ \times 0.41 \\ \hline 3012 \\ 12048 \\ \hline 1.23492 \end{array}$$

SELF-CORRECTING EXERCISE F, page 19.

2. $0.9_{\wedge}\overline{)\,11.0_{\wedge}}$

3. $5.21_{\wedge}\overline{)\,0.18\overset{\wedge}{4}}$

4. $0.002_{\wedge}\overline{)\,14.800_{\wedge}}$

5. (a) $8.225 \div 0.05 = 164.5$

   (b) $2.622 \div 5.7 = 0.46$

END OF LESSON 4

LESSON RECORD FORM

0902 Math 9
Revised 11/90

FOR STUDENT USE ONLY

Date Lesson Submitted

_____

Time Spent on Lesson

_____

(If label is missing or incorrect)

File Number

_____

Lesson Number _____

FOR A.D.L.C. USE ONLY

Assigned Teacher: _____

Lesson Grading: _____

Additional Grading E/R/P Code: _____

Mark: _____

Graded by: _____

Assignment Code: _____

Date Lesson Received:

_____

Lesson Recorded _____

Student's Questions and Comments

Apply Lesson Label Here

Name

Address

Postal Code

Please verify that preprinted label is for correct course and lesson.

Teacher's Comments:

_____

Correspondence Teacher

St. Serv. 14-91

ALBERTA DISTANCE LEARNING CENTRE

MAILING INSTRUCTIONS FOR CORRESPONDENCE LESSONS

1. BEFORE MAILING YOUR LESSONS, PLEASE SEE THAT:

(1) All pages are numbered and in order, and no paper clips or staples are used.
(2) All exercises are completed. If not, explain why.
(3) Your work has been re-read to ensure accuracy in spelling and lesson details.
(4) The Lesson Record Form is filled out and the correct lesson label is attached.
(5) This mailing sheet is placed on the lesson.

2. POSTAGE REGULATIONS

Do **not** enclose letters with lessons.

Send all letters in a separate envelope.

3. POSTAGE RATES

First Class

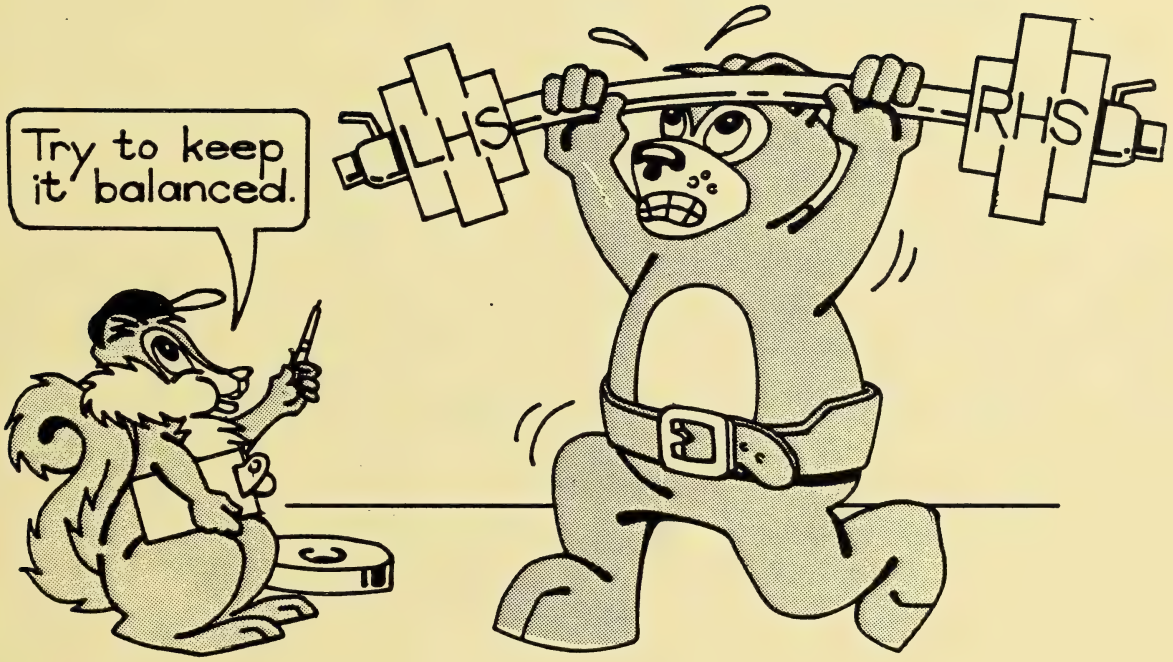
Take your lesson to the Post Office and have it weighed. Attach sufficient postage and a green first-class sticker to the front of the envelope, and seal the envelope. Correspondence lessons will travel faster if first-class postage is used.

Try to mail each lesson as soon as it has been completed.

When you register for correspondence courses, you are expected to send lessons for correction regularly. Avoid sending more than two or three lessons in one subject at the same time.

St. Serv. 9-91

SOLVING EQUATIONS



In this lesson you will learn about:

- solving equations of the type:

$x + a = b$
$ax = b$
$ax + c = b$
$ax + bx = c$
$ax + b = cx + d$

KEEPING SKILLFUL

EXERCISE 1

1. Beside each number, write its opposite (additive inverse).

(a) 6 _____ (b) -12 _____

(c) $-\frac{3}{4}$ _____ (d) 2.5 _____

2. Beside each number, write its reciprocal (multiplicative inverse).

(a) $\frac{2}{3}$ _____ (b) $-\frac{4}{5}$ _____

(c) 8 _____ (d) $-2\frac{1}{4}$ _____

3. Fill in the blanks to make the following statements true.

(a) $9 + \_\_\_\_ = 0$ (b) $-5 + \_\_\_\_ = 0$

(c) $\frac{7}{8} \times \_\_\_\_ = 1$ (d) $-\frac{3}{5} \times \_\_\_\_ = 1$

4. Perform the following operations.

(a) $-9 + 6 =$ _____ (b) $4 - (-12) =$ _____

(c) $-18 \div -2 =$ _____ (d) $-3 \times 6 =$ _____

EQUATION SOLVING

An equation is a statement that two mathematical expressions are equal. To keep the left-hand side (L.H.S.) of an equation equal to the right-hand side (R.H.S), you must always perform the same operation to both sides.

The objective in solving an equation is to obtain the variable by itself on one side of the equation. To do this, you will have to "undo" the operations that are being performed on that variable. Recall that opposite operations "undo" each other.

Addition and subtraction are opposite operations.

Division and multiplication are opposite operations.

Page 1 lists the different types of equations that will be solved in this lesson. For each of these types, the letter x represents the variable and the letters, a, b, c, and d represent rational numbers.

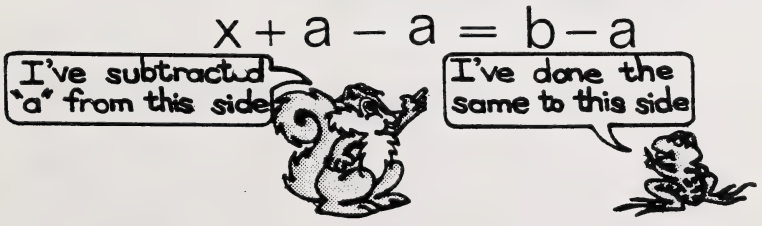
EQUATIONS OF THE TYPE $x + a = b$

If a number is added to the variable, you can undo the operation by subtracting that number, However, you must remember, what you do to one side of an equation you must also do to the other side.

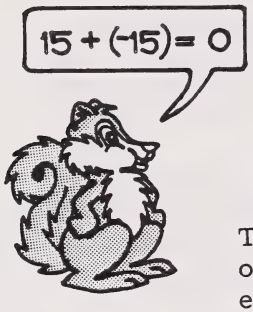
From your work with integers in Lesson 1 you should recall that subtracting a number is the same as adding its opposite.

Rule

> To solve an equation where a number is added to the variable, add the opposite of that number to both sides of the equation.



Example 1 Solve $x + 15 = 32$.



$$x + 15 = 32$$

$$x + 15 + (-15) = 32 + (-15)$$ *Add -15 to both sides. -15 is the opposite of 15.*

$$x = 17$$

To check the solution, substitute the answer into the original equation. If the left-hand side of the equation equals the right-hand side, the solution is correct.

Check:

L.H.S.	R.H.S.
$x + 15$	32
$17 + 15$	
32	✓

Since L.H.S. = R.H.S., the solution, 17, is correct.

Example 2 Solve $-9 + x = 41$.

$$-9 + x = 41$$

$$-9 + 9 + x = 41 + 9$$

$$x = 50$$

Check:

L.H.S	R.H.S.
$-9 + x$	41
$-9 + 50$	
41	✓

The solution is $x = 50$.

Example 3 Solve $x - 25 = 4$

Recall that subtracting 25 is the same as adding -25. Thus, the opposite of -25 is +25.

$$x - 25 = 4$$

$$x - 25 + 25 = 4 + 25$$

$$x = 29$$

Check:

L.H.S.	R.H.S.
$x - 25$	4
$29 - 25$	
4	✓

The solution is $x = 29$.

SELF-CORRECTING EXERCISE A

Solve: Show your solution steps and do the checks.

1. (a) $n + 8 = 15$ Check:

L.H.S.	R.H.S.

(b) $2 = z + 1$ Check:

L.H.S.	R.H.S.

(c) $7 + x = -4$ Check:

L.H.S.	R.H.S.

(d) $x - 2 = -3$ Check:

L.H.S.	R.H.S.

Check your answers with those on page 17 of this lesson.

EXERCISE 2 Solve the following equations. Show your solution steps and do the checks.

1. $b + 12 = -3$ Check: L.H.S. | R.H.S.

2. $x - 22 = 6$ Check: L.H.S. | R.H.S.

3. $-33 = x - 9$ Check: L.H.S. | R.H.S.

4. $-2 + y = 7$ Check: L.H.S. | R.H.S.

EQUATIONS OF THE TYPE $ax = b$

Earlier in this lesson it was mentioned that multiplication and division are inverse (opposite) operations. In some equations the variable is multiplied by a number. To "undo" this multiplication, divide both sides of the equations by that number. Study page 112 of *Holt Mathematics 3*.

Reference

Example 1 Solve $6x = -54$.

Divide both sides by 6.

$$6x = -54$$

$$\frac{6x}{6} = \frac{-54}{6}$$

$$x = -9$$

Check:

L.H.S.	R.H.S.
$6x$	-54
$6(-9)$	
-54	✓

When you were working with fractions you learned that dividing by a number is the same as multiplying by its reciprocal. This is a good rule to remember when solving equations that involve fractions.

Example 2 Solve $-\frac{2}{3}x = 8$.

Instead of dividing by $-\frac{2}{3}$, multiply by the reciprocal, $-\frac{3}{2}$.

$$-\frac{2}{3}x = 8$$

$$-\frac{3}{2} \times -\frac{2}{3}x = \frac{4}{1} \times -\frac{3}{2}$$

$$1x = -12$$

$$x = -12$$

Check:

L.H.S.	R.H.S.
$-\frac{2}{3}x$	8
$-\frac{2}{3}(-12)$	
$-\frac{2}{3} \times -\frac{12}{1}$	
$+8$	✓

Rule

> To solve an equation where the variable is multiplied by a number:
>
> - divide both sides of the equation by that number, or
> - multiply both sides of the equation by the reciprocal of that number.

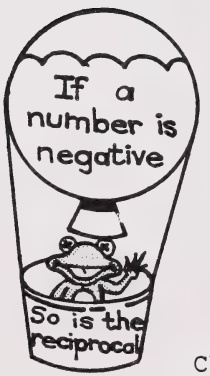
SELF-CORRECTING EXERCISE B

Solve: Show your solution steps and do the checks.

1. (a) $4z = -4$ Check:

(b) $-y = 3$ Check:

(c) $\frac{2}{3}h = -4$ Check:



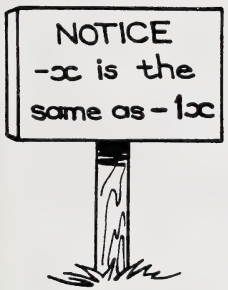
(d) $\frac{-1}{4}a = -2$ Check:

Check your answers with those on page 18 of this lesson.
Correct any errors in your work.

EXERCISE 3 Solve the following equations. Show your solution steps and do the checks.

1. $5x = -105$ Check:

2. $\frac{3}{4}x = 27$ Check:



3. $-x = 22$ Check:

4. $-\frac{1}{2}x = \frac{3}{8}$ Check:

EQUATIONS OF THE TYPE $ax+c=b$

Sometimes several operations are being done to a variable in an equation. If a variable is being multiplied by a number, and a number is being added to that product, the rules on pages 3 and 7 of this lesson must both be applied. Apply these rules in the following order.

Rule

To solve an equation of the type $ax + c = b$, where a, c, and b are any rational numbers:

1. First add the opposite of c to both sides of the equation.

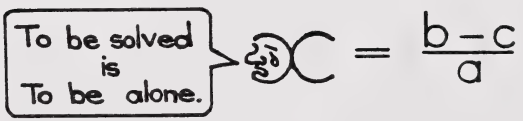
$$ax + c - c = b - c$$

$$ax = b - c$$

2. Then, divide both sides of the equation by a.

$$\frac{ax}{a} = \frac{b - c}{a}$$

$$x = \frac{b - c}{a}$$



Example Solve $3x + 2 = -7$.

	$3x + 2 = -7$
Add -2 to both sides.	$3x + 2 - 2 = -7 - 2$
	$3x = -9$
Divide both sides by 3.	$\frac{3x}{3} = \frac{-9}{3}$
	$x = -3$

Check:

L.H.S.	R.H.S.
$3x + 2$	-7
$3(-3) + 2$	
$-9 + 2$	
-7	✓

SELF-CORRECTING EXERCISE C

Solve the following equations. Show the solution steps and do the checks.

1. $20 = 4 + 8x$ Check:

2. $5 - x = -31$ Check:

   (Note: 5 - x means 5 added to -x)

Check your answers with those on page 19 of this lesson. Correct any errors in your work.

EXERCISE 4

Solve the following equations. Show the solution steps and do the checks.

1. $\frac{2}{3}x + 1 = 9$ Check:

2. $18 - x = 22$ Check:

EQUATIONS OF THE TYPE $ax + bx = c$

Consider the equation $2x + 3x = 20$. You know that two items of one type plus three items of the same type equal five items of that type. Similarly, two x's plus three x's equal five x's.

The equation $2x + 3x = 20$ can be simplified to $5x = 20$. You solved equations of this type, $ax = b$, on page 7 of this lesson.

Example 1 Solve $2x + 3x = 20$.

$$2x + 3x = 20$$
$$5x = 20$$
$$\frac{5x}{5} = \frac{20}{5}$$
$$x = 4$$

Check:

L.H.S.	R.H.S.
$2x + 3x$	20
$2(4) + 3(4)$	
$8 + 12$	
20	✓

It is important to realize that the equation $2x + 3 = 20$ is not the same as the one given above. $2x + 3$ means two x's plus the whole number 3.

$$x + x + 3 = ?$$

The expression $2x + 3$ cannot be simplified. Equations of this type were solved on page 10 of this lesson.

Example 2 Solve $-9 = 4x - x$.

$-x$ is equal to $-1x$.

$$-9 = 4x - 1x$$
$$-9 = 3x$$
$$\frac{-9}{3} = \frac{3x}{3}$$
$$-3 = x$$

Check:

L.H.S.	R.H.S.
-9	$4x - 1x$
	$4(-3) - 1(-3$
	$-12 + 3$
✓	-9

SELF-CORRECTING EXERCISE D

Solve the following equations. Show your solution steps and do the checks.

1. $26 = -15x + 2x$ Check:

2. $-11x - x = 9$ Check:

Check your answers with those on page 19 and 20 of this lesson. Correct any errors in your work.

EXERCISE 5 Solve the following equations. Show your solution steps and do the checks.

1. $-8x + 2x = 42$ Check:

2. $-14 = x + 6x$ Check:

EQUATIONS OF THE TYPE $cx+b = cx+d$

Reference

Study Example 2 on page 108 and Example 1 on page 115 of *Holt Mathematics 3.*

Example 1

Consider the equation $2x = 5x + 6$. To solve this equation, the terms containing the variable (2x and 5x) must be brought together on the same side of the equation. This can be done by adding the opposite of one of these terms to both sides.

$$2x = 5x + 6$$

$$2x - 5x = 5x - 5x + 6$$

By adding -5x to both sides, the variable terms are brought together on the L.H.S. of the equation.

$$-3x = 6$$

$$\frac{-3x}{-3} = \frac{6^{2}}{-3}$$

$$x = -2$$

Example 2

To further complicate matters, you could also have number terms on both sides of the equation. An example is $9x - 5 = 6 - 2x$. By the process of adding opposites, the number terms can be brought to one side of the equation and the variable terms to the other side.

$$9x - 5 = 6 - 2x$$

$$9x + 2x - 5 = 6 - 2x + 2x$$

Add 2x to both sides to bring the variable terms together on the L.H.S.

$$11x - 5 = 6$$

$$11x - 5 + 5 = 6 + 5$$

Add 5 to both sides to bring the number terms together on the R.H.S.

$$11x = 11$$

$$\frac{11x}{11} = \frac{11}{11}$$

$$x = 1$$

Check:

L.H.S.	R.H.S.
9x - 5	6 - 2x
9(1) - 5	6 - 2(1)
9 - 5	6 - 2
4	4 ✓

SELF-CORRECTING EXERCISE E

Solve: Show your solution steps and do the checks.

1. (a) $3x - 2 = 18 - x$ Check:

(b) $2y - 3 - 6y = -35$ Check:

(c) $\frac{1}{2}a + 4 = a - 1$ Check:

Check your answers with those on page 20 and 21 of this lesson. Correct any errors in your work.

EXERCISE 6 Solve the following equations. Show your solution steps and be sure to do the checks.

1. $9x - 4 = -10 + 7x$ Check:

2. $-5 - 6x = 2x + 35$ Check:

3. $2x + 9 + 3x = x - 3$ Check:

KEY TO SELF-CORRECTING EXERCISES IN LESSON 5

SELF-CORRECTING EXERCISE A, page 5.

1. (a) $N + 8 = 15$

$N + 8 - 8 = 15 - 8$

$N = 7$

This is the same as writing $N + 8 + (-8) = 15 + (-8)$.

Check:	L.H.S.	R.H.S.
	$N + 8$	15
	$7 + 8$	
	15	✓

(b) $2 = z + 1$

$2 - 1 = z + 1 - 1$

$1 = z$

Notice that the variable can be on either side of the equation.

Check:	L.H.S.	R.H.S.
	2	$z + 1$
		$1 + 1$
	✓	2

(c) $7 + x = -4$

$7 - 7 + x = -4 - 7$

$x = -11$

With practice you may subtract $7 - 7$ in your head and simply write $x = -4 - 7$, $x = -11$.

Check:	L.H.S.	R.H.S.
	$7 + x$	-4
	$7 + (-11)$	
	-4	✓

(d) $x - 2 = -3$

$x - 2 + 2 = -3 + 2$

$x = -1$

Check:	L.H.S.	R.H.S.
	$x - 2$	-3
	$-1 - 2$	
	-3	✓

SELF-CORRECTING EXERCISE B, page 8.

1. (a) $4z = -4$

$$\frac{4z}{4} = \frac{-4}{4}$$

$$z = -1$$

Check:	L.H.S.	R.H.S.
	$4z$	-4
	$4(-1)$	
	-4	✓

(b) $-x = 3$

Notice that $-x$ is the same as $-1x$.

$$\frac{-1x}{-1} = \frac{3}{-1}$$

$$x = -3$$

Check:	L.H.S.	R.H.S.
	$-x$	3
	$-(-3)$	
	$+3$	✓

(c) $\frac{2}{3}h = -4$

$$\frac{\cancel{3}^{1}}{\cancel{2}_{1}} \times \frac{\cancel{2}_{1}}{\cancel{3}_{1}}h = \frac{-\cancel{4}^{2}}{1} \times \frac{3}{\cancel{3}_{1}}$$

$$h = -6$$

Check:	L.H.S.	R.H.S.
	$\frac{2}{3}h$	-4
	$\frac{2}{3}(-6)$	
	$\frac{\cancel{2}_{1}}{\cancel{3}_{1}} \times \frac{-\cancel{6}^{2}}{1}$	
	-4	✓

(d) $-\frac{1}{4}a = -2$

$$\frac{-\cancel{4}^{1}}{1} \times \frac{-1}{\cancel{4}_{1}}a = \frac{-2}{1} \times \frac{-4}{1}$$

$$a = +8$$

Check:	L.H.S.	R.H.S.
	$-\frac{1}{4}a$	-2
	$-\frac{1}{4}(8)$	
	$-\frac{1}{\cancel{4}_{1}} \times \frac{\cancel{8}^{2}}{1}$	
	-2	✓

SELF-CORRECTING EXERCISE C, page 11.

1. $20 = 4 + 8x$

$20 - 4 = 4 - 4 + 8x$

$16 = 8x$

$\frac{16}{8} = \frac{8x}{8}$

$2 = x$

This equation could be written as $4 + 8x = 20$.

Check:	L.H.S.	R.H.S.
	20	$4 + 8x$
		$4 + 8(2)$
		$4 + 16$
	✓	20

2. $5 - x = -31$

$5 - 1x = -31$

$5 - 5 - 1x = -31 - 5$

$-1x = -36$

$\frac{-1x}{-1} = \frac{-36}{-1}$

$x = +36$

Check:	L.H.S.	R.H.S.
	$5 - x$	-31
	$5 - (+36)$	
	$5 + (-36)$	
	-31	✓

Note: When the variable is not multiplied by a number, assume the number is 1. x means 1x and -x means -1x.

SELF-CORRECTING EXERCISE D, page 13.

1. $26 = -15x + 2x$

$26 = -13x$

$\frac{26}{-13} = \frac{-13x}{-13}$

$-2 = x$

Check:	L.H.S.	R.H.S.
	26	$-15x + 2x$
		$-15(-2) + 2(-2)$
		$+30 + (-4)$
	✓	+26

SELF-CORRECTING EXERCISE D, page 13 (continued).

2. $-11x - x = 9$

$-11x - 1x = 9$

$-12x = 9$

$\frac{-12x}{-12} = \frac{9}{-12}$

$x = \frac{-3}{4}$

Check:

L.H.S.	R.H.S.
$-11x - x$	9
$-11(-\frac{3}{4}) - (-\frac{3}{4})$	
$\frac{+33}{4} + \frac{3}{4}$	
$\frac{36}{4}$	
9	✓

SELF-CORRECTING EXERCISE E, page 15.

(a) $3x - 2 = 18 - x$

Remember:
-x is the same as -1x.

$3x - 2 = 18 - 1x$

$3x + 1x - 2 = 18 - 1x + 1x$

$4x - 2 = 18$

$4x - 2 + 2 = 18 + 2$

$4x = 20$

$\frac{4x}{4} = \frac{20}{4}$

$x = 5$

Check:

L.H.S.	R.H.S.
$3x - 2$	$18 - x$
$3(5) - 2$	$18 - 5$
$15 - 2$	13
13	✓

SELF-CORRECTING EXERCISE E, page 15 (continued).

(b) $2y - 3 - 6y = -35$

$2y - 6y - 3 = -35$

$-4y - 3 = -35$

$-4y - 3 + 3 = -35 + 3$

$-4y = -32$

$\frac{-4y}{-4} = \frac{-32}{-4}$

$y = +8$

Since both variable terms are already on the same side of the equation, you simply add them.

Check:	L.H.S.	R.H.S.
	$2y - 3 - 6y$	-35
	$2(8) - 3 - 6(8)$	
	$16 - 3 - 48$	
	$13 - 48$	
	-35	✓

(c) $\frac{1}{2}a + 4 = a - 1$

$\frac{1}{2}a - 1a + 4 = 1a - 1a - 1$

$-\frac{1}{2}a + 4 = -1$

$-\frac{1}{2}a + 4 - 4 = -1 - 4$

$-\frac{1}{2}a = -5$

$\frac{-2}{1} \times \frac{-1}{2}a = -5 \times \frac{-2}{1}$

$a = 10$

Check:	L.H.S.	R.H.S.
	$\frac{1}{2}a + 4$	$a - 1$
	$\frac{1}{2}(10) + 4$	$10 - 1$
	$5 + 4$	9
	9	✓

END OF LESSON 5

LESSON RECORD FORM

0902 Math 9
Revised 11/90

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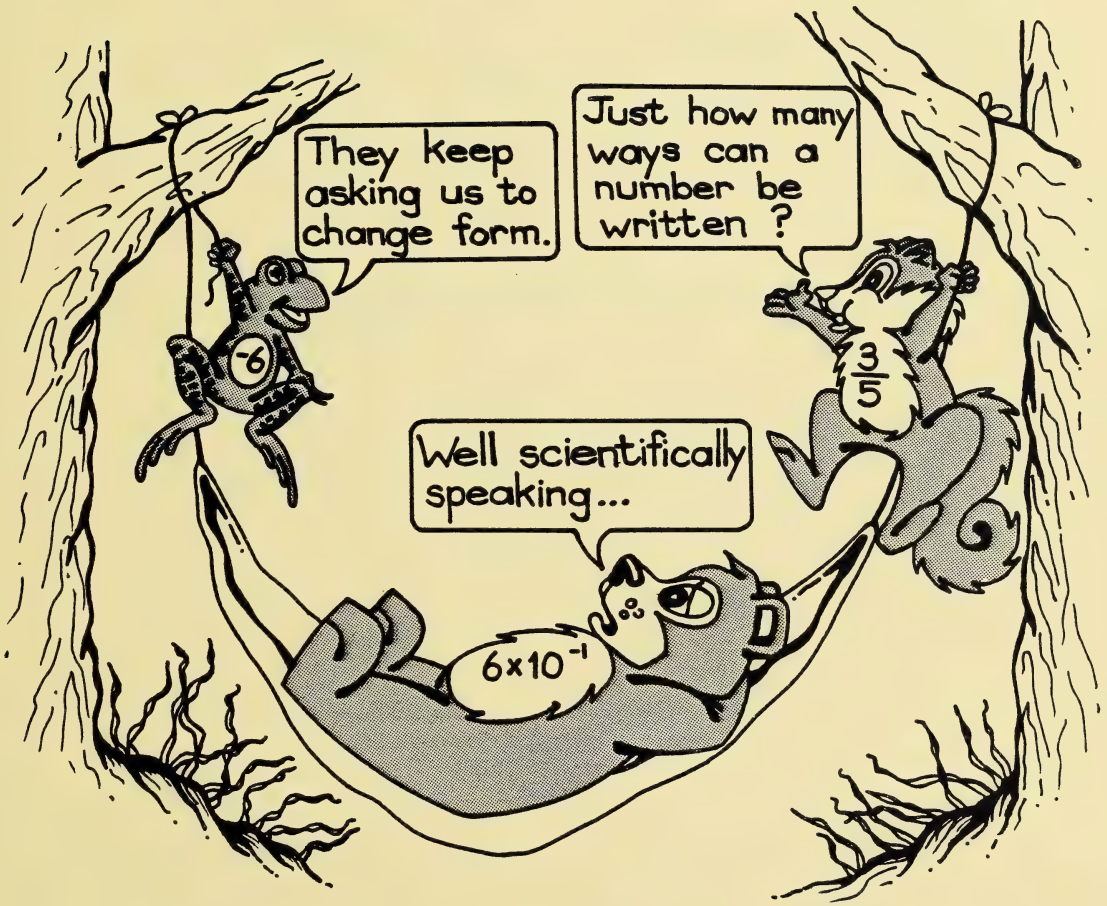
FORMS OF WRITING RATIONAL NUMBERS



In this lesson you will learn about:

- multiplying and dividing by powers of ten
- writing numbers in scientific notation
- changing fractions to decimals
- changing decimals to fractions

KEEPING SKILLFUL

EXERCISE 1 1. Evaluate the following powers.

(a) 2^4 = _____ (b) 2^{-4} = _____

(c) $(-9)^3$ = _____ (d) 8^{-2} = _____

(e) $(\frac{1}{3})^2$ = _____ (f) 9^0 = _____

2. Divide the following decimals until they repeat or terminate.

(a) $1.4\overline{)7.5068}$ (b) $0.12\overline{)53}$

POWERS OF TEN

Reference For a review of exponential notation read pages 52 and 60 of *Holt Mathematics 3.*

Study the following chart of powers of ten.

POWERS OF TEN

Decimal Form	⟷	Exponential Form
10 000	$10 \times 10 \times 10 \times 10$	10^4
1 000	$10 \times 10 \times 10$	10^3
100	10×10	10^2
10	10	10^1
1	1	10^0
0.1	$\frac{1}{10}$	10^{-1}
0.01	$\frac{1}{100}$ or $\frac{1}{10^2}$	10^{-2}
0.001	$\frac{1}{1000}$ or $\frac{1}{10^3}$	10^{-3}

MULTIPLICATION USING POWERS OF TEN

Examples

$24 \times 1000 = 24 \times 10^3 = 24\,000$ ← *Multiplying by 10^3 moved the decimal 3 places to the right.*

$24 \times 100 = 24 \times 10^2 = 2\,400$

$24 \times 0.01 = 24 \times 10^{-2} = 0.24$

$24 \times 0.001 = 24 \times 10^{-3} = 0.024$ ← *Multiplying by 10^{-3} moved the decimal 3 places to the left.*

Short-Cut Rule

When multiplying by a power of ten:

- a positive exponent indicates the number of places to move the decimal to the right.
- a negative exponent indicates the number of places to move the decimal to the left.

DIVISION USING POWERS OF TEN

Examples

$24 \div 1000 = 24 \div 10^3 = 0.024$ ← *Dividing by 10^3 moved the decimal 3 places to the left.*

$24 \div 100 = 24 \div 10^2 = 0.24$

$24 \div 0.01 = 24 \div 10^{-2} = 2\ 400$

$24 \div 0.001 = 24 \div 10^{-3} = 24\ 000$ ← *Dividing by 10^{-3} moved the decimal 3 places to the right.*

Short-Cut Rule

When dividing by a power of ten:

- a positive exponent indicates the number of places to move the decimal to the left.
- a negative exponent indicates the number of places to move the decimal to the right.

EXERCISE 2 1. Do the following operations using the short-cut rules.

(a) $0.4 \times 10^4 =$ _____ (c) $11.2 \div 10^3 =$ _____

(b) $39 \times 10^{-3} =$ _____ (d) $0.084 \div 10^{-5} =$ _____

2. Do the following operations. It may be helpful to change the powers of ten to exponential form.

(a) $8.6 \times 1000 =$ $8.6 \times 10^3 = 8600$ (c) $8 \times 100\ 000 =$ _____

(b) $0.005 \times 0.01 =$ _____ (d) $0.9 \times 10 =$ _____

SCIENTIFIC NOTATION

In scientific notation, a number is written as a power of ten times a number, x, such that $1 \leq x < 10$.

Reference Read page 64 of *Holt Mathematics 3.*

Examples The following numbers have been changed to scientific notation.

(a) 6 400 000 = 6.4×10^{6}

(b) 20 900 = 2.09×10^{4}

(c) 102 = 1.02×10^{2}

(d) 0.000 403 = 4.03×10^{-4}

(e) 0.01 = 1×10^{-2}

(f) 0.000 029 = 2.9×10^{-5}

Notice that the decimal always comes after the first non-zero number.

To change a number to scientific notation, simply move the decimal from its present position so that it comes after the first non-zero digit. Choose the exponent of the power of ten to express the number of places the decimal was moved. If the decimal was moved to the left, as in 6 400 000, the power will be positive. If the decimal was moved to the right, as in 0.000 403 the power will be negative.

To change a number from scientific notation to its usual decimal form, simply apply the rules on page 4 of this lesson.

$4.03 \times 10^{-4} = 0.000\ 403$ *Multiplying by a negative power moves the decimal to the left.*

$6.4 \times 10^{6} = 6\ 400\ 000$ *Multiplying by a positive power moves the decimal to the right.*

EXERCISE 3 1. Write the following numbers in scientific notation.

(a) 13.5 = _____

(b) 0.07 = _____

(c) 0.000 043 2 = _____

(d) 583 200 000 = _____

(e) 43 600 = _____

2. Write the following numbers in decimal form.

(a) 9.2×10^4 = _____

(b) 3.08×10^{-6} = _____

(c) 7×10^6 = _____

(d) 2.084×10^{-2} = _____

(e) 6.54×10^1 = _____

CHANGING FRACTIONS TO DECIMALS

Recall that a fraction is another form of showing division.

$\frac{1}{2}$ means $1 \div 2$.

When a fraction has a denominator which is a multiple of 10, the short-cut rule on page 4 of this lesson can be applied.

Example 1

$$\frac{53}{100} = 53 \div 100$$

$$= 53 \div 10^2$$ ← *Dividing by 10^2 moves the decimal 2 places to the left.*

$$= 0.53$$

When a fraction has a denominator which is not a multiple of 10, divide the numerator by the denominator.

Example 2 $\frac{5}{8} = 5 \div 8$

$= 0.625$ *This decimal terminates.*

Example 2 $\frac{2}{7} = 2 \div 7$

$= 0.\dot{2}85714\dot{2}$ *This decimal repeats.*

In Lesson 4 you learned how to divide with decimal numbers. At that time, you also found out that the quotients would either terminate or repeat. The same holds true for changing fractions to decimals.

Rule

> Any fraction can be expressed as either a terminating or repeating decimal.

Reference Read page 22 of *Holt Mathematics 3.*

SELF-CORRECTING EXERCISE A

Write the following as decimals.

1. (a) $\frac{7}{10}$ (b) $\frac{37}{1000}$

   (c) $\frac{23}{10}$ (d) $\frac{2314}{1000}$

2. Change into decimal form.

   (a) $\frac{83}{25}$ (b) $\frac{11}{24}$

Check your answers with those on page 19 of this lesson.

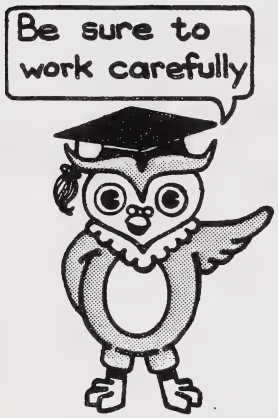
EXERCISE 4 Change the following to decimals. Show your work for Questions 3, 4, 5 and 6.

1. $\frac{98}{10\ 000}$ = _____

2. $\frac{32}{10}$ = _____

3. $\frac{15}{16}$ = _____

4. $\frac{33}{40}$ = _____



5. $\frac{5}{12}$ = _____

6. $\frac{9}{11}$ = _____

CHANGING TERMINATING DECIMALS TO FRACTIONS

Reference Study Examples 1 and 2 on page 23 of *Holt Mathematics 3*. You may also wish to refer to pages 10 - 12 of Lesson 4 for a review of place values.

Example Write 0.327 as a fraction.

Method 1

$$0.327 = 327 \times 10^{-3}$$

The decimal was moved so that 327 is written as a whole number.

$$= 327 \times \frac{1}{10^3}$$

$$= 327 \times \frac{1}{1000}$$

$$= \frac{327}{1000}$$

Method 2 A simpler approach is to let the place value of the last digit determine the denominator of the fraction.

$$0.327 = \frac{327}{1000}$$

The place value of the last digit is "thousandths".

Rule

> To write a decimal number as a fraction:
>
> - let the numerator equal the digits in the decimal number.
> - let the denominator equal the place value of the last digit in the decimal number.

Warning When applying the above rule, never put a decimal point in the numerator.

Example 2 Write 9.42 as a fraction in lowest terms.

$$9.42 = \frac{\cancel{942}\ 471}{\cancel{100}\ 50} = \frac{471}{50}$$

SELF-CORRECTING EXERCISE B

Write the following as fractions in lowest terms.

1. (a) 0.25 (b) 0.2

(c) 0.125 (d) 1.8

Check your answers with those on page 19 of this lesson.

EXERCISE 5 Write the following as fractions in lowest terms.

1. (a) 3.09 (b) 0.0004

(c) 23.5 (d) 0.625

CHANGING REPEATING DECIMALS TO FRACTIONS

Reference

Study Examples 3 and 4 on page 23 of *Holt Mathematics 3.* There is an error in Example 3. The second line should read $0.\dot{3} = \frac{1}{3}$.

Example 1

Consider the repeating decimal $0.\dot{4}\dot{7}$. Recall that $0.\dot{4}\dot{7}$ is the same as 0.474747. . . Multiplying by 100 will bring one complete repeating block in front of the decimal point.

Let A represent 0.474747. . .

Then, $100A = 47.4747\ldots$ *Multiplying by 100 moves the decimal 2 places to the right.*

Now think - what can you subtract from 47.4747. . . so that all the digits after the decimal point are eliminated?

The answer is 0.4747. . . This is what 1A represents.

Take 100A and subtract 1A.

$$\begin{aligned} 100A &= 47.4747\ldots \\ -1A &= 0.4747\ldots \\ \hline 99A &= 47.0000\ldots \end{aligned}$$

or, $99A = 47$

By solving for A, you will obtain the fraction equivalent to $0.\dot{4}\dot{7}$.

$$99A = 47$$

$$A = \frac{47}{99}$$

Using this method you can change any repeating decimal to a fraction.

Example 2 Change $9.\dot{2}4\dot{1}$ to a fraction.

Let A = 9.241241. . .

Multiplying by 1000 will bring one complete repeating block in front of the decimal point.

$$\begin{aligned} 1000A &= 9241.241\ldots \\ -1A &= \quad 9.241\ldots \\ \hline 999A &= 9232 \end{aligned}$$

Subtracting 1A eliminated the digits after the decimal point.

$$A = \frac{9232}{999}$$

SELF-CORRECTING EXERCISE C

1. Change $4.\dot{1}$ to a fraction.

Check your answer with the one on page 19 of this lesson. Correct any errors in your work.

EXERCISE 6 1. Change $0.\dot{6}$ to a fraction. Show all your work.

2. Change $8.\dot{2}\dot{3}$ to a fraction. Show all your work.

Short-cut Method

In a repeating decimal, the repeating block is called the repetend. To express a repeating decimal as a fraction, use the repetend as the numerator of the fraction and a numeral, made up of as many 9's as there are digits in the repetend, as the denominator. Reduce this fraction to lowest terms to express the decimal as a fraction in simplest form.

Examples

$0.\overline{3}333\ldots = \frac{3}{9} = \frac{1}{3}$

1 nine in the denominator.

1 digit in the repetend.

$0.\overline{69}69\ldots = \frac{69}{99} = \frac{23}{33}$

2 nines in the denominator.

2 digits in the repetend.

$0.\overline{321}321\ldots = \frac{321}{999} = \frac{107}{333}$

3 nines in the denominator.

3 digits in the repetend.

$$31.\dot{7}\dot{2} = 31 + 0.\overline{72}72\ldots$$

$$= 31 + \frac{72}{99}$$

$$= 31\frac{8}{11}$$

$$= \frac{349}{11}$$

Mixed number changed to an improper fraction.

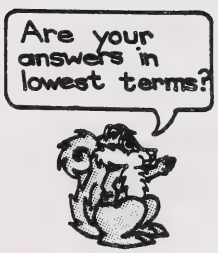
EXERCISE 7 Using the short-cut method, write the following as fractions in lowest terms.

1. $0.729729\ldots = \frac{\overline{729}}{999} =$

2. $2.0303\ldots = 2\frac{\overline{03}}{99} =$

3. $0.123123\ldots =$

4. $3.1515\ldots =$



5. $5.3939\ldots =$

CHANGING REPEATING DECIMALS TO FRACTIONS

Sometimes in a repeating decimal there are digits that come after the decimal point but are not in the repeating block.

For example: $0.8\dot{9}\dot{4}$ *The 8 comes after the decimal point but is not a part of the repetend.*

Example 1 Change $0.8\dot{9}\dot{4}$ to a fraction.

Multiplying by 1000 will bring one complete repetend in front of the decimal point.

Let A equal 0.89494. . .

Then, 1000A = 894.9494. . .

Now think - what multiple of A can you subtract from 1000A so that all the digits after the decimal point are eliminated?

Perhaps 1A:

$$\begin{array}{rl} 1000A = & 894.9494\ldots \\ -1A = & \quad 0.89494\ldots \\ \hline \end{array}$$

Since the digits after the decimal point are not identical, this will not work.

Try 10A:

$$\begin{array}{rl} 1000A = & 894.9494\ldots \\ -10A = & \quad 8.9494\ldots \\ \hline 990A = & 886 \end{array}$$

The digits after the decimal are identical, so this works.

$$A = \frac{886}{990}$$

$$A = \frac{443}{495}$$

Notice that there was 1 digit after the decimal point that was not in the repetend and the multiple of A that was subtracted had 1 zero.

Example 2 Change $26.12\dot{3}$ to a fraction.

Let A represent 26.123333. . .

$$\begin{aligned} 1000A &= 26123.333\ldots \\ -100A &= 2612.333\ldots \\ \hline 900A &= 23511 \end{aligned}$$

$$A = \frac{23511}{900}$$

$$A = \frac{7837}{300}$$

Again, notice that there were 2 digits after the decimal point that were not in the repetend and the multiple of A that was subtracted had 2 zeros.

SELF-CORRECTING EXERCISE D

1. Write $9.3\dot{8}\dot{4}$ as a fraction in lowest terms. Show your work.

Check your answer with the one on page 19 of this lesson. Correct any errors in your work.

The short-cut method given on page 13 of this lesson, *cannot* be used when digits other than the repetend follow the decimal point.

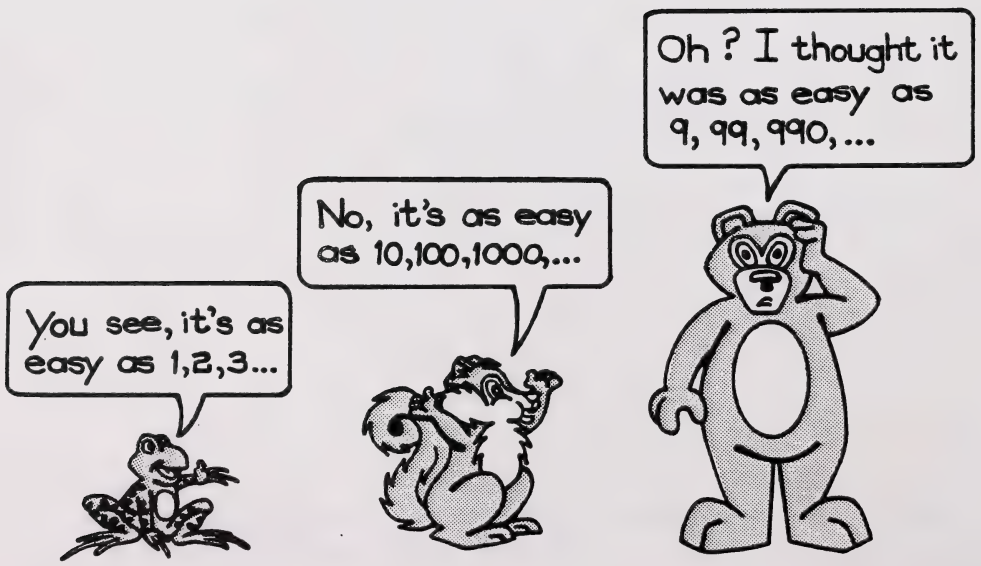
EXERCISE 8 Write the following as fractions in lowest terms. Show your work.

1. $0.8\dot{1}6\dot{3}$

2. $32.15\dot{6}\dot{7}$

EXERCISE 8 (continued)

3. $5.3\dot{7}8\dot{9}$



KEY TO SELF-CORRECTING EXERCISES IN LESSON 11

SELF-CORRECTING EXERCISE A, Page 7.

1. (a) $\frac{7}{10} = 0.7$ (b) $\frac{37}{1000} = 0.037$ (c) $\frac{23}{10} = 2.3$ (d) $\frac{2314}{1000} = 2.314$

2. (a)
$$\begin{array}{r} 3.32 \\ 25\overline{)83.000} \\ \underline{75} \\ 80 \\ \underline{75} \\ 50 \\ \underline{50} \\ 0 \end{array}$$

(b)
$$\begin{array}{r} .458\dot{3} \\ 24\overline{)11.0000} \\ \underline{96} \\ 140 \\ \underline{120} \\ 200 \\ \underline{192} \\ 80 \\ \underline{72} \\ 8 \end{array}$$

SELF-CORRECTING EXERCISE B, Page 10.

1. (a) $0.25 = \frac{25}{100} = \frac{1}{4}$ (b) $0.2 = \frac{2}{10} = \frac{1}{5}$

(c) $0.125 = \frac{125}{1000} = \frac{1}{8}$ (d) $1.8 = 1\frac{8}{10} = 1\frac{4}{5}$

SELF-CORRECTING EXERCISE C, Page 12.

1. Let A represent 4.111...

$$10A = 41.11...$$
$$-1A = \ \ 4.11...$$
$$9A = 37$$
$$A = \frac{37}{9}$$

SELF-CORRECTING EXERCISE D, Page 16.

1. Let A represent 9.38484...

$$1000A = 9384.84...$$
$$-10A = \ \ 93.84...$$
$$990A = 9291$$
$$A = \frac{9291}{990}$$
$$= \frac{3097}{330}$$

END OF LESSON 6

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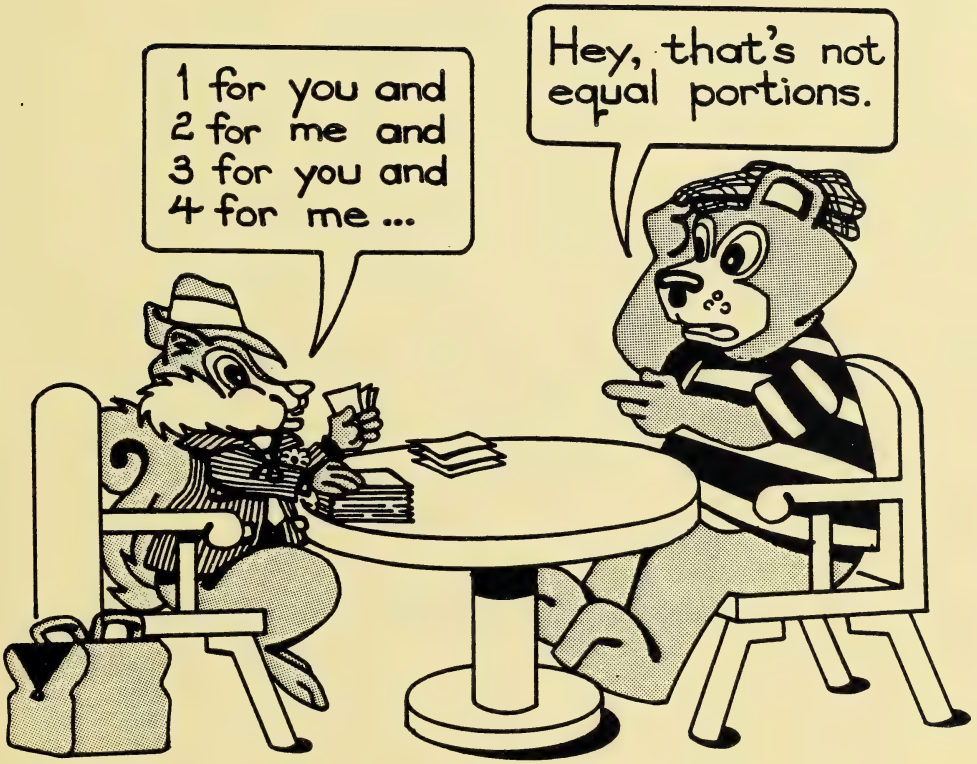
RATIO and PROPORTION



In this lesson you will learn about:

- proportions using whole and rational numbers
- fractions, percents and decimals
- solving the three types of percent problems

KEEPING SKILLFUL

EXERCISE 1

1. Write the following numbers in scientific notation.

   (a) 86 900 000 = _____

   (b) 0.000 025 = _____

   (c) 824 000 000 000 = _____

   (d) 0.038 = _____

2. Write the following numbers in decimal form.

   (a) 8.3×10^{5} = _____

   (b) 3.8×10^{-6} = _____

   (c) 2.084×10^{-3} = _____

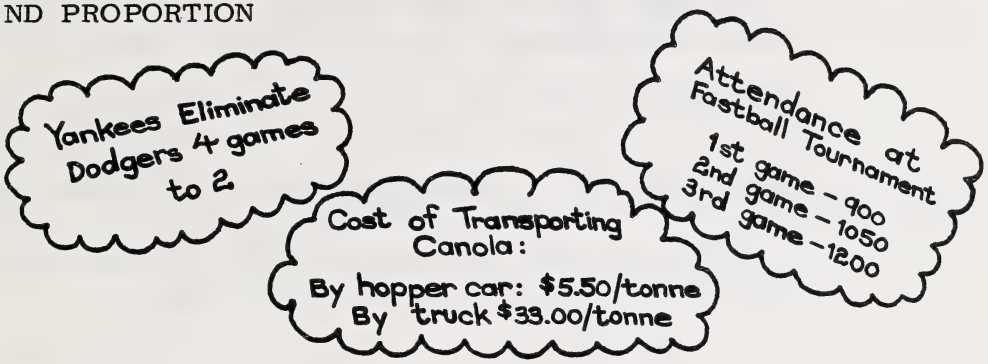
   (d) 5.1×10^{8} = _____

3. Write the following as fractions.

   (a) 0.85

   (b) $0.17\dot{3}$

RATIO AND PROPORTION


Yankees Eliminate Dodgers 4 games to 2.
Attendance at Fastball Tournament
1st game - 900
2nd game - 1050
3rd game - 1200
Cost of Transporting Canola:
By hopper car: $5.50/tonne
By truck $33.00/tonne

In each of the above statements, numbers are compared. A comparison of two or more numbers is called a ratio.

The above statements would then have the following ratios:

4 : 2

Each number is called a term of the ratio.

$5.50 : $33.00

This symbol is read "to". It compares the numbers.

900 : 1050 : 1200

Reference

Read page 139 of *Holt Mathematics 3.* Try to do some of the questions on pages 139 and 140 of your own. Check your results with the answers provided at the back of the textbook.

Proportion

A proportion is a statement that two ratios are equivalent. Sometimes the ratios are referred to as being equivalent fractions.

An example could be:

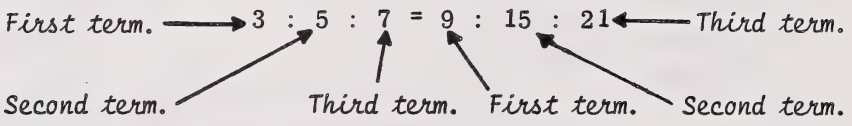
First term. → 2 : 1 = 4 : 2 ← *Second term.*

Second term. ↑ *First term.* ↑

or, this may be written as:

$$\frac{2}{1} = \frac{4}{2}$$

Another example could be written where more than two terms are compared.



When you write these proportions in fractional form, remember to always work from left to right.

So, $\frac{3}{5} = \frac{9}{15}$, $\frac{5}{7} = \frac{15}{21}$, and $\frac{3}{7} = \frac{9}{21}$.

Note that the first term is compared to the second term, or the first term is compared to the third term or the second term is compared to the third term. You must compare the same terms in the same order on both sides of the equals sign.

Reference Read page 143 of *Holt Mathematics 3* very carefully.

EXERCISE 2 1. Write all the possible fractional equivalents for each of the following proportions.

(a) 5 : 10 = 15 : 30

(b) b : 5 = 20 : c

(c) 1 : 5 : 7 = 10 : 50 : 70

$\frac{1}{5} = \frac{10}{50}$; $\frac{5}{7} = \frac{50}{70}$; $\frac{1}{7} = \frac{10}{70}$

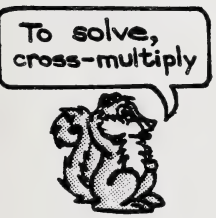
(d) h : 4 : 16 = 2 : 8 : g

Reference Study Examples 1 to 3 on page 143 of *Holt Mathematics 3*. Then, continue working on Exercise 2.

EXERCISE 2 (continued)

2. Solve the following. The variables are from the set of Whole Numbers.

(a) 2 : 5 = 10 : x



(b) 3 : 5 = x : 95

(c) 1 : b : c = 9 : 36 : 45

$\frac{1}{b} = \frac{9}{36}$ $\quad$ $\frac{1}{c} = \frac{9}{45}$

$9b = 36$ $\quad$ $9c = 45$

$b = 4$ $\quad$ $c = 5$

(d) x : 8 : 9 = 12 : y : 36

EXERCISE 2 (continued)

3. Solve the following. The variables are from the set of **Rational Numbers.** Express your answer in the form $\frac{a}{b}$, $b \neq 0$.

(a) $7 : y = 3 : 10$

(b) $a : b : 7 = 5 : 8 : 10$

(c) $9 : 4 = x : 7$

(d) $3 : 4 : y = x : 20 : 25$

APPLICATIONS

Knowing how to work with proportions is helpful when you want to solve problems involving comparisons. Usually in a ratio type problem you are given three items and you are asked to find the fourth item.

Reference Study Example 1 on page 146 of *Holt Mathematics 3.*

Example Given the ratio 5 : 3 : 1, find an equivalent ratio whose sum is 90.

In order to find these numbers, you must, first of all, find the sum of the parts of the ratio.

$$5 + 3 + 1 = 9$$

Let x, y, z represent the numbers, respectively.

$\frac{5}{9} = \frac{x}{90}$ $\frac{3}{9} = \frac{y}{90}$ $\frac{1}{9} = \frac{z}{90}$

$9x = 450$ $9y = 270$ $9z = 90$

$x = 50$ $y = 30$ $z = 10$

The new numbers are 50, 30, and 10.

EXERCISE 3 1. Three numbers are in the ratio 1 : 2 : 3. If their sum is 72, find the numbers.

EXERCISE 3 (continued)

2. At the local grocery store Jean saw a sale sign advertising tomato soup at 3 cans for 98¢. If she has only $2.50, how many cans can she buy?

3. A soil mixture has loam, sand and clay in the ratio 4 : 3 : 2.

   (a) How much loam is needed to make 450 kg of this soil mixture?

   (b) How much sand will be required?

FRACTIONS AND PERCENT

Percent Percent means "over 100". Thus, 23% means $\frac{23}{100}$.

Changing Fractions to Percents

> To change a fraction to a percent, you must find an equivalent fraction using 100 as the new denominator.

Example 1 Write $\frac{3}{5}$ as a percent.

$$\frac{3}{5} = \frac{x}{100}$$

$$5x = 300$$

$$x = \frac{300}{5}$$

$$x = 60$$

Therefore, $\frac{3}{5} = \frac{60}{100}$ or 60%.

Short Cut

> A short-cut is often used when changing a fraction to a percent. Simply multiply the numerator by 100% and divide by the denominator.

Example 2 Change $\frac{3}{5}$ to a percent.

$$\frac{3}{5} = \frac{3 \times 100\%}{5} = \frac{300\%}{5} = 60\%$$

However, to change a mixed number to a percent, first change it to an improper fraction.

Changing Percents to Fractions

Since percent means "over 100", to change a percent to a fraction, you would place the percent over 100 and then reduce to lowest terms.

Example 1 Change 125% to a fraction or mixed number.

$125\% = \frac{125}{100}$ ← Place the % over 100

$= \frac{5}{4}$ ← Reduce

$= 1\frac{1}{4}$ ← Change the improper fraction to a mixed number.

Example 2 Change 87.5% to a fraction.

$87.5\% = \frac{87.5}{100}$ ← Place the % over 100

$= \frac{87.5 \times 10}{100 \times 10}$ ← Multiply both the numberator and denominator by 10 to eliminate the decimal fraction.

$= \frac{875}{1000}$

$= \frac{7}{8}$ ← Reduce

Example 3 Change $66\frac{2}{3}$ % to a fraction.

$66\frac{2}{3}\% = \frac{200}{3}\%$ ← Change the mixed number to an improper fraction.

$\frac{200}{3}\% = \frac{\frac{200}{3}}{100}$ ← Place the % over 100

$= \frac{200}{3} \times \frac{1}{100}$ ← Change division to multiplication.

$= \frac{2}{3}$ ← Reduce

EXERCISE 4

1. Change the following fractions to percents.

(a) $\frac{4}{5}$ (b) $\frac{7}{12}$

(c) $\frac{3}{2}$ (d) $1\frac{3}{4}$

2. Change each of the following percents to a fraction in lowest terms or a mixed number.

(a) 99% (b) 17.2%

(c) $5\frac{1}{4}\%$ (d) 520%

PERCENT AND DECIMALS

Reference

To express a decimal as a percent, move the decimal point two places to the right. To express a percent as a decimal move the decimal point two places to the left.

Study page 150 of *Holt Mathematics 3.*

On page 150 of your textbook one example is 15% = 0.15. The reason for this is:

$$15\% = \frac{15}{100}$$ ← *% means "over 100".*

$$= 0.15$$

Also, $172\% = \frac{172}{100} = 1.72.$

Rule

> To change a percent to a decimal, move the decimal point two places to the left and remove the percent sign.

To change a decimal to a percent, the reverse procedure is followed.

Examples

$$0.38 = \frac{38}{100} = 38\%$$

The decimal moved 2 places to the right.

$$0.175 = \frac{17.5}{100} = 17.5\%$$

Rule

> To change a decimal to a percent, move the decimal point two places to the right and include the percent sign.

EXERCISE 5 1. Express the following decimals as a percent.

(a) 0.452 = _____

(b) 2.13 = _____

(c) 3.5 = _____

(d) 0.008 = _____

(e) 100 = _____

2. Express the following percents as a decimal.

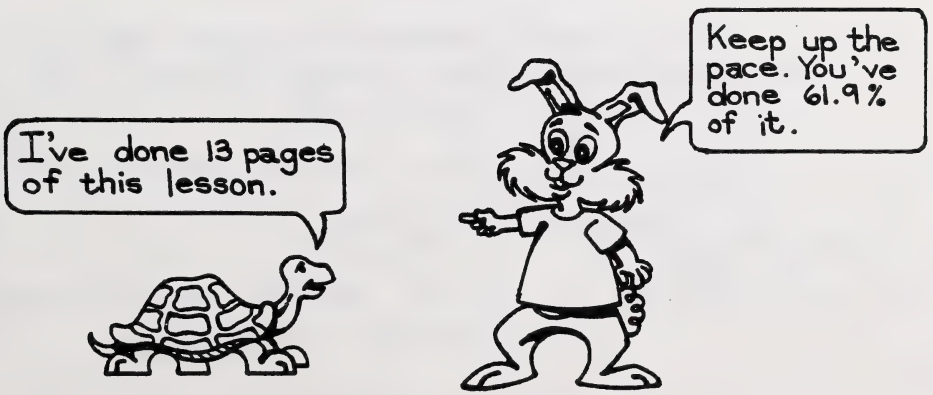
(a) 345% = _____

(b) 15.17% = _____

(c) 0.052% = _____

(d) 89% = _____

(e) 95.00% = _____



PROBLEMS INVOLVING PERCENTS

In problems involving percents, there are three basic variables a, b, and r. A proportion is set up as follows.

$$\frac{a}{b} = \frac{r}{100}$$

In the above proportion the three variables are:

- a, which is the amount and is the numerator of the first ratio.
- b, which is the base and is the denominator of the first ratio.
- r, which is the rate and is placed over 100. Together, r and 100 form the second ratio.

All percent problems can be changed into a sentence of the form:

a is r% of b

The base is usually preceded by the word "of".

From the above statement you can see that there are only three different types of percent problems. They are:

1. ___?___ is r% of b. *When the amount is not given.*

2. a is ___?___ % of b. *When the rate is not given.*

3. a is r% of ___?___. *When the base is not given.*

Example 1 What is 30% of \$800?

First identify a, b, and r. Write your problem in the form:

a is r% of b. Note: the base is proceded by the word "of".

In this case: $\underset{\uparrow a}{\_\_\_\_}$ is $\underset{\uparrow r}{30\%}$ of $\underset{\uparrow b}{\$800}$.

Use this information to write the proportion $\frac{a}{b} = \frac{r}{100}$.

$$\frac{a}{800} = \frac{30}{100}$$

Since the percent is less then 100%, the amount must be less than the base.

$$a \times 100 = 30 \times 800$$

$$a = \frac{30 \times \overset{8}{\cancel{800}}}{\underset{1}{\cancel{100}}}$$

$$a = 240$$

Thus, \$240 is 30% of \$800.

Reference For more examples in finding the amount, refer to page 151 of *Holt Mathematics 3*.

Example 2 What percent of 25 is 80?

$\underset{\uparrow a}{80}$ is $\underset{\uparrow r}{?\%}$ of $\underset{\uparrow b}{25}$

$$\frac{80}{25} = \frac{r}{100}$$

Since the amount is larger than the base, "r" will be larger than 100%.

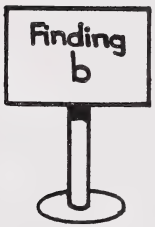
$$25r = 80 \times 100$$

$$r = 320$$

So, 80 is 320% of 25.

Reference For more examples in finding the rate, refer to page 153 of *Holt Mathematics 3*.

Example 3 186 is 75% of what number?



$$\frac{186}{a} \text{ is } \frac{75\%}{r} \text{ of } \frac{?}{b}$$

$$\frac{186}{b} = \frac{75}{100}$$

$$75b = 186 \times 100$$

$$b = \frac{\cancel{186}^{62} \times \cancel{100}^{4}}{\cancel{75}_{1}\cancel{3}}$$

$$b = 248$$

Thus, 186 is 75% of 248.

Reference For more examples in finding the base, turn to page 155 of *Holt Mathematics 3*.

SELF-CORRECTING EXERCISE A

1. For each of the following, put the problem into the general sentence form "a is r% of b". (Leave a blank for the unknown quantity.)

(a) What is 5% of 93? ____ is 5% of 93

(b) 71% of what number is 152? _____

(c) 34 is what percent of 19? _____

(d) 23% of 68 is what number? _____

2. Using each of the following problems expressed in the general sentence form "a is r% of b", set up the ratio $\frac{a}{b} = \frac{r}{100}$.

(a) 63 is r% of 29. $\frac{63}{29} = \frac{r}{100}$

(b) a is 17% of 138. _____

(c) 44 is 32% of b. _____

(d) 51 is r% of 17. _____

Check your answers with those on page 21 of this lesson. Correct any errors in your work.

EXERCISE 6 1. For each of the following, put the problem into the general sentence "a is r% of b". Leave a blank for the unknown quantity.

(a) 920 is what percent of 180? _____

(b) 128% of 62 is what number? _____

(c) What is $12\frac{1}{2}$% of 25? _____

(d) 900% of what number is 450? _____

EXERCISE 6 (continued)

2. For each of the following questions, write the general sentence "a is r% of b". Then write its ratio in the form $\frac{a}{b} = \frac{r}{100}$. Do not solve the questions.

(a) In British Columbia, the sales tax is 7% of the selling price. If a color television sells for $550.00, how much sales tax must be paid?

Sentence _____ Ratio _____

(b) Dave saved 10% of his income. If Dave was able to save $832.50 each year, what was his income?

Sentence _____ Ratio _____

(c) The Tow and Go ski shop was having their annual spring sale. If the discount was 20%, how much could Gary expect to save on a $375 pair of skis?

Sentence _____ Ratio _____

Here is a general procedure you can use to solve percentage problems.

Problem Solving Method

1. Carefully read through the problem.
2. Analyze the problem - put it into the general form (a is r% of b) to identify the base, rate, and amount.
3. Use the proportion $\frac{a}{b} = \frac{r}{100}$ to solve for the unknown.
4. Use a sentence to relate the answer to the question.

Example

Ron Green paid \$2368.50 in income tax last year. The rate at which he paid was 20% of his gross income. How much did he earn last year?

"of" indicates that his gross income is the base.

$$\underset{\uparrow}{\underset{a}{\$2368.50}} \text{ is } \underset{\uparrow}{\underset{r}{20\%}} \text{ of } \underset{\uparrow}{\underset{b}{\$\_\_\_\_}}.$$

$$\frac{2368.50}{b} = \frac{20}{100}$$

$$b \times 20 = 2368.50 \times 100$$

$$\frac{b \times \cancel{20}^{1}}{\cancel{20}_{1}} = \frac{2368.50 \times \cancel{100}^{5}}{\cancel{20}_{1}}$$

$$b = 11\ 842.50$$

Ron's gross income was \$11 842.50.

EXERCISE 7 Applications. Show the solution steps as shown on page 18.

1. Jonathan bought a new suit for \$198.00. If the sales tax was 6%, what was the amount of tax to be paid on the suit?

2. Lydia's salary for working at the Red Grill Restaurant is $750.00 per month. She also received $225.00 in tips for the month of June. The amount received in tips was what percentage of her total earnings?

   Hint: Calculate her total earnings before doing anything else.

3. There were 11 850 people at a recent hockey game at the Coliseum. This was 72% of the total capacity. Find the total capacity.

KEY TO SELF-CORRECTING EXERCISES IN LESSON 9

SELF-CORRECTING EXERCISE A, pages 16 and 17.

1. (a) 152 is 71% of _____

   (b) 34 is _____ % of 19

   (c) _____ is 23% of 68

2. (b) $\frac{a}{138} = \frac{17}{100}$

   (c) $\frac{44}{b} = \frac{32}{100}$

   (d) $\frac{51}{17} = \frac{r}{100}$

END OF LESSON 7

LESSON RECORD FORM

0902 Math 9
Revised 11/90

FOR STUDENT USE ONLY

Date Lesson Submitted

_____

Time Spent on Lesson

_____

(If label is missing or incorrect)

File Number

_____

Lesson Number _____

FOR A.D.L.C. USE ONLY

Assigned Teacher: _____

Lesson Grading: _____

Additional Grading E/R/P Code: _____

Mark: _____

Graded by: _____

Assignment Code: _____

Date Lesson Received:

_____

Lesson Recorded _____

Student's Questions and Comments

Apply Lesson Label Here

Name

Address

Postal Code

Please verify that preprinted label is for correct course and lesson.

Teacher's Comments:

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Correspondence Teacher

St. Serv. 14-91

ALBERTA DISTANCE LEARNING CENTRE

MAILING INSTRUCTIONS FOR CORRESPONDENCE LESSONS

1. BEFORE MAILING YOUR LESSONS, PLEASE SEE THAT:

(1) All pages are numbered and in order, and no paper clips or staples are used.
(2) All exercises are completed. If not, explain why.
(3) Your work has been re-read to ensure accuracy in spelling and lesson details.
(4) The Lesson Record Form is filled out and the correct lesson label is attached.
(5) This mailing sheet is placed on the lesson.

2. POSTAGE REGULATIONS

Do not enclose letters with lessons.

Send all letters in a separate envelope.

3. POSTAGE RATES

First Class

Take your lesson to the Post Office and have it weighed. Attach sufficient postage and a green first-class sticker to the front of the envelope, and seal the envelope. Correspondence lessons will travel faster if first-class postage is used.

Try to mail each lesson as soon as it has been completed.

When you register for correspondence courses, you are expected to send lessons for correction regularly. Avoid sending more than two or three lessons in one subject at the same time.

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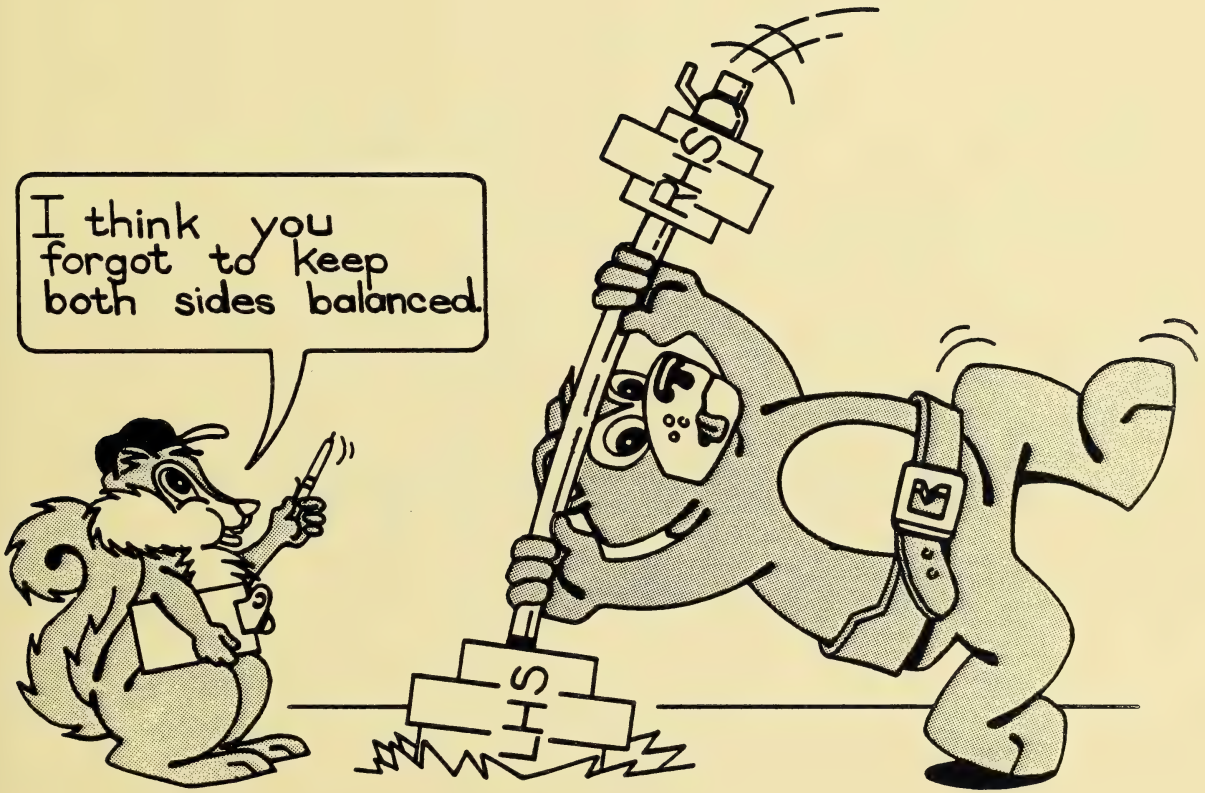
SOLVING EQUATIONS and WORD PROBLEMS



In this lesson you will learn about:

- solving equations of the form

$$\frac{x}{a} = b \text{ and } \frac{x}{a} = \frac{b}{c}$$

- solving equations using the distributive property and LCM
- solving word problems

KEEPING SKILLFUL

EXERCISE 1

1. Find the LCM of the following sets of numbers. Show your work.

   (a) 24, 42

   (b) 18, 20, 28

2. Perform the indicated operations. Show your work. Express your answers in lowest terms.

   (a) $2\frac{3}{8} + \frac{1}{5}$

   (b) $9\frac{2}{3} - 6\frac{1}{2}$

   (c) $1\frac{3}{4} \times \frac{4}{9}$

   (d) $2\frac{1}{5} \div 1\frac{2}{3}$

REVIEW OF PREVIOUS EQUATION SOLVING

In Lesson 5 several types of equations were solved. Here is a review of these types.

1. To solve equations of the type $x + a = b$, add the opposite of a to both sides.

   Example:

$$x + 6 = -2$$
$$x + 6 + (-6) = -2 + (-6)$$
$$x = -8$$

2. To solve equations of the type $ax = b$, divide both sides by a (or, multiply both sides by the reciprocal of a).

   Examples:

   (a) $-4x = 36$

$$\frac{-4x}{-4} = \frac{36}{-4}$$
$$x = -9$$

   (b) $\frac{2}{3}x = 12$

$$\frac{3}{2} \times \frac{2}{3}x = \frac{12}{1} \times \frac{3}{2}$$
$$x = 18$$

3. To solve equations of the type $ax + b = c$, first add the opposite of b to both sides, then divide both sides by a.

   Example:

$$3x - 4 = 5$$
$$3x - 4 + 4 = 5 + 4$$
$$3x = 9$$
$$\frac{3x}{3} = \frac{9}{3}$$
$$x = 3$$

4. To solve equations of the type $ax + bx = c$, first add the variable terms ax and bx together.

Example:

$$-7x + 5x = 27$$

$$-2x = 27$$

$$\frac{-2x}{-2} = \frac{27}{-2}$$

$$x = -13.5$$

5. To solve equations of the type $ax + b = cx + d$, bring the variable terms together on one side and the number terms together on the other side. To do this use the process of adding opposites.

Example:

$$3x - 4 = 5x + 8$$

$$3x - 5x - 4 = 5x - 5x + 8$$ ← *Add $-5x$ to both sides.*

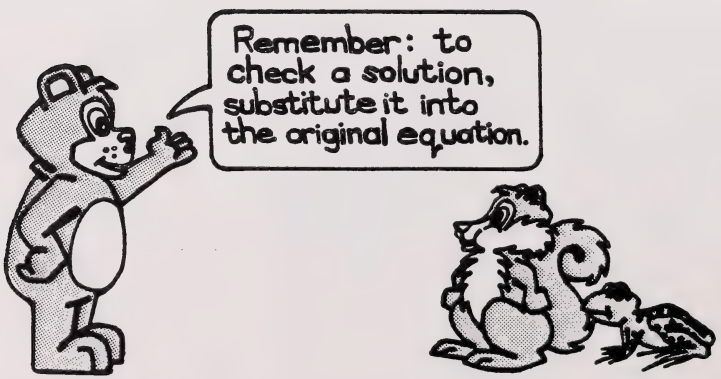
$$-2x - 4 = 8$$

$$-2x - 4 + 4 = 8 + 4$$ ← *Add $+4$ to both sides.*

$$-2x = 12$$

$$\frac{-2x}{-2} = \frac{12}{-2}$$

$$x = -6$$



EXERCISE 2 Solve the following equations. Show your solution steps and do the checks.

1. $-9x = 153$ Check:

L.H.S.	R.H.S.
$-9x$	153

2. $-6 + x = 23$ Check:

3. $25 = \frac{5}{7}x$ Check:

4. $5 - 4x = -27$ Check:

5. $8x + 12 = -2x - 18$ Check:

THE DISTRIBUTIVE PROPERTY

Given that a, b and c are any rational numbers, then $a(b + c) = (a \times b) + (a \times c)$.

Notice that a is multiplied by both b and c.

Examples

$-2(x + 3) = (-2 \times x) + (-2 \times 3) = -2x - 6$

$5(x - 4) = (5 \times x) + (5 \times -4) = 5x - 20$

$-(2x + 3) = (-1 \times 2x) + (-1 \times 3) = -2x - 3$

The main thing to remember about the distributive property is that the term outside the brackets is multiplied by both terms inside the brackets. The operation inside the brackets can be either addition or subtraction.

Exercise 3 Simplify each of the following using the distributive property.

(a) $5(x + 3) =$ _____

(b) $3(2x + 5) =$ _____

(c) $4(3x - 5) =$ _____

(d) $-2(3x - 7) =$ _____

(e) $-(6x - 5) =$ _____

SOLVING EQUATIONS USING THE DISTRIBUTIVE PROPERTY

Example 1 Solve $3(x + 4) = 16$.

Distributive Property

$$3(x + 4) = 16$$
$$3x + 12 = 16$$
$$3x + 12 - 12 = 16 - 12$$
$$3x = 4$$
$$\frac{3x}{3} = \frac{4}{3}$$
$$x = \frac{4}{3}$$

Check:

L.H.S.	R.H.S.
$3(x + 4)$	16
$3(\frac{4}{3} + 4)$	
$3(\frac{4}{3}) + 3(4)$	
$4 + 12$	
16	✓

Notice that using the distributive property removed the brackets. The resulting equation is of the form $ax + b = c$ which was covered earlier.

Reference Study Example 3 on page 109 of *Holt Mathematics 3.*

SELF-CORRECTING EXERCISE A

Solve:

1. $4(5n - 7) = 10n - 4 + 9n$ Check:

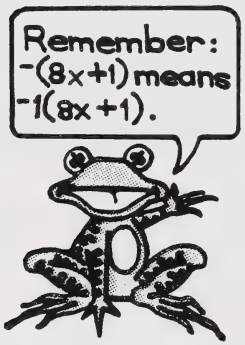
2. $3(6a - 2) = 4(4a + 1) + a$ Check:

Check your answers with those on page 17 of this lesson.
Correct any errors in your work.

EXERCISE 4 Solve the following equations. Show your solution steps and do the checks.

1. $-3(2x - 4) = 24$ Check:

2. $-(8x + 1) = 15$ Check:



3. $-5(x - 6) + 2(3x + 7) = 4$ Check:

SOLVING EQUATIONS OF THE TYPE $\frac{x}{a} = b$

In Lesson 5 you learned that multiplication and division are opposite operations which "undo" each other. Therefore, to solve an equation where a variable is being divided by a number, multiply both sides by that number.

Rule

> To solve equations of the form $\frac{x}{a} = b$, multiply both sides by a.

Example 1 Solve $\frac{x}{4} = 13$.

$$\frac{x}{4} = 13$$

$$\frac{4}{1} \times \frac{x}{4} = \frac{13}{1} \times \frac{4}{1}$$

$$x = 52$$

Check:

L.H.S.	R.H.S.
$\frac{x}{4}$	13
$\frac{52}{4}$	
13	✓

SOLVING EQUATIONS OF THE TYPE $\frac{x}{a} = \frac{b}{c}$

Often cross-multiplication is used to solve equations of this type. Recall that to cross multiply, you should multiply each numerator by the denominator on the other side of the equation.

Example 5 Solve $\frac{2x}{-3} = \frac{24}{9}$.

$$\frac{2x}{-3} = \frac{24}{9}$$

Cross-multiply.

$$9(2x) = 24(-3)$$

$$18x = -72$$

Divide both sides by 18.

$$\frac{\cancel{18}^{1}x}{\cancel{18}_{1}} = \frac{\cancel{-72}^{-4}}{\cancel{18}_{1}}$$

$$x = -4$$

Check:

L.H.S.	R.H.S.
$\frac{2x}{-3}$	$\frac{24}{9}$
$\frac{2(-4)}{-3}$	$\frac{\cancel{24}^{8}}{\cancel{9}_{3}}$
$\frac{-8}{-3}$	$\frac{8}{3}$
$\frac{+8}{3}$	✓

Example 2 Solve $\frac{3x + 3}{6} = \frac{-2x + 1}{-2}$.

Cross Multiply

$$-2(3x + 3) = 6(-2x + 1)$$

Use the distributive property

$$-6x - 6 = -12x + 6$$

$$-6x + 12x - 6 = -12x + 12x + 6$$

$$6x - 6 = 6$$

$$6x - 6 + 6 = 6 + 6$$

$$6x = 12$$

$$\frac{\cancel{6}^{1}x}{\cancel{6}_{1}} = \frac{\cancel{12}^{2}}{\cancel{6}_{1}}$$

$$x = 2$$

Check:

L.H.S.	R.H.S.
$-2(3x + 3)$	$6(-2x + 1)$
$-2[3(2) + 3]$	$6[-2(2) + 1]$
$-2[6 + 3]$	$6[-4 + 1]$
$-2(9)$	$6(-3)$
-18	-18 ✓

SELF-CORRECTING EXERCISE B

Solve the following equations. Show your solution steps and do the checks.

1. $\frac{x + 9}{4} = -7$ Check:

2. $\frac{2 - x}{5} = \frac{x - 2}{4}$ Check:

Check your answers with those on pages 19 and 20 of this lesson. Correct any errors in your work.

EXERCISE 5 Solve the following equations. Show your solution steps and do the checks.

1. $\frac{x - 7}{8} = \frac{x - 1}{6}$ Check:

2. $\frac{9x - 5}{13} = 1$ Check:

3. $\frac{x - 3}{3} = \frac{3 - x}{10}$ Check:

SOLVING EQUATIONS BY FINDING THE LCM

If you have forgotten how to find the least common multiple (LCM), of two or more numbers, review Lesson 3.

Consider the equation, $\frac{8x}{7} - \frac{3}{14} = -\frac{5}{2}$. You cannot solve this equation by cross-multiplication because there are two rational expressions, $\frac{8x}{7}$ and $\frac{3}{14}$, on the L.H.S. Instead, you can remove all of the denominators from the equation by multiplying both sides by the LCM of the three denominators.

Example 1 Solve the equation, $\frac{8x}{7} - \frac{3}{14} = -\frac{5}{2}$.

The LCM of 7, 14 and 2 is 14.

Multiply both sides by 14.

$$14\left(\frac{8x}{7} - \frac{3}{14}\right) = \left(-\frac{5}{2}\right)14$$

Use the distributive property.

$$\frac{\cancel{14}^{2}}{1}\left(\frac{8x}{\cancel{7}_{1}}\right) - \frac{\cancel{14}^{1}}{1}\left(\frac{3}{\cancel{14}_{1}}\right) = \left(-\frac{5}{\cancel{2}_{1}}\right)\frac{\cancel{14}^{7}}{1}$$

Notice that this equation is now of the type $ax + b = c$.

$$16x - 3 = -35$$

$$16x - 3 + 3 = -35 + 3$$

$$16x = -32$$

$$\frac{\cancel{16}^{1}x}{\cancel{16}_{1}} = \frac{\cancel{-32}^{2}}{\cancel{16}_{1}}$$

$$x = -2$$

Check:

L.H.S.	R.H.S.
$\frac{8x}{7} - \frac{3}{14}$	$-\frac{5}{2}$
$\frac{8(-2)}{7} - \frac{3}{14}$	
$\frac{-16}{7} - \frac{3}{14}$	
$\frac{-32}{14} - \frac{3}{14}$	
$\frac{\cancel{-35}^{5}}{\cancel{14}_{2}}$	
$-\frac{5}{2}$	✓

Reference Study Example 3 on page 115 of *Holt Mathematics 3*. Notice that the Alternate Method using the LCM is simpler.

Example 2 Solve $\frac{5x - 3}{2} = \frac{9x}{6} - \frac{x + 12}{4}$.

$$12\left(\frac{5x - 3}{2}\right) = 12\left(\frac{9x}{6} - \frac{x + 12}{4}\right)$$

The LCM of 2, 6, and 4 is 12.

$$\frac{\cancel{12}^{6}}{1}\left(\frac{5x - 3}{\cancel{2}_{1}}\right) = \frac{\cancel{12}^{2}}{1}\left(\frac{9x}{\cancel{6}_{1}}\right) - \frac{\cancel{12}^{3}}{1}\left(\frac{x + 12}{\cancel{4}_{1}}\right)$$

Multiplying by -3 changed this sign.

$$30x - 18 = 18x - 3x - 36$$

$$30x - 18 = 15x - 36$$

$$30x - 15x = -36 + 18$$

$$15x = -18$$

$$x = -\frac{\cancel{18}^{6}}{\cancel{15}_{5}}$$

$$x = -1\frac{1}{5}$$

SELF-CORRECTING EXERCISE C

Solve.

1. (a) $\frac{4-y}{5} - \frac{2-3y}{2} = \frac{y+51}{20}$

Check your answers with those on page 18 of this lesson. Correct any errors in your work.

EXERCISE 6 Solve the following equations. Show your solution steps.

1. $\frac{2x}{3} + \frac{1}{5} = \frac{2}{15}$

2. $\frac{4 - a}{5} - \frac{2 - 3a}{2} = \frac{a + 51}{10}$

3. $\frac{2 - a}{3} = \frac{a + 1}{2} + \frac{a - 2}{3}$

EXERCISE 7 Solve the following equations. Show your solution steps.

1. $4a + 2 = 6$

2. $\frac{x + 1}{2} = \frac{3}{5}$

3. $\frac{b}{2} - \frac{b}{5} = 6$

4. $\frac{y - 2}{3} - \frac{2y - 1}{4} = \frac{1}{2}$

Key to Self-Correcting Exercises in Lesson 6

Self-Correcting Exercise A, page 7.

1. $4(5n - 7) = 10n - 4 + 9n$

$20n - 28 = 19n - 4$

$20n - 19n - 28 = 19n - 19n - 4$

$1n - 28 = -4$

$n - 28 + 28 = -4 + 28$

$n = 24$

Check:

L.H.S.	R.H.S.
$4(5n - 7)$	$10n + 4 + 9n$
$4[5(24) - 7]$	$10(24) - 4 + 9(24)$
$4[120 - 7]$	$240 - 4 + 216$
$4(113)$	452
452	✓

2. $3(6a - 2) = 4(4a + 1) + a$

$18a - 6 = 16a + 4 + 1a$

$18a - 6 = 17a + 4$

$18a - 17a - 6 = 17a - 17a + 4$

$1a - 6 = 4$

$1a - 6 + 6 = 4 + 6$

$a = 10$

Check:

L.H.S.	R.H.S.
$3(6a - 2a)$	$4(4a + 1) + a$
$3[6(10) - 2]$	$4[4(10) + 1] + 10$
$3(60 - 2)$	$4(41) + 10$
$3(58)$	$164 + 10$
174	174

Self-Correcting Exercise B, Page 11.

1. $\frac{x - 9}{4} = -7$

$4\left(\frac{x + 9}{4}\right) = -7(4)$

$x + 9 = -28$

$x + 9 - 9 = -28 - 9$

$x = -37$

Check:

L.H.S.	R.H.S.
$\frac{x + 9}{4}$	-7
$\frac{-37 + 9}{4}$	
$\frac{-28}{4}$	
-7	✓

SELF-CORRECTING EXERCISE B, page 11 (continued).

2. $\frac{2 - x}{5} = \frac{x - 2}{4}$

$4(2 - x) = 5(x - 2)$

$8 - 4x = 5x - 10$

$8 - 4x - 5x = 5x - 5x - 10$

$8 - 9x = -10$

$8 - 8 - 9x = -10 - 8$

$-9x = -18$

$\frac{-9x}{-9} = \frac{-18}{-9}$

$x = 2$

Check:

L.H.S.	R.H.S.
$\frac{2 - x}{5}$	$\frac{x - 2}{4}$
$\frac{2 - 2}{5}$	$\frac{2 - 2}{4}$
$\frac{0}{5}$	$\frac{0}{4}$
0	0 ✓

SELF-CORRECTING EXERCISE C, page 14.

1. (e) $\frac{4 - y}{5} - \frac{2 - 3y}{2} = \frac{y + 51}{20}$

$\frac{20}{1}\left(\frac{4 - y}{5}\right) - \frac{20}{1}\left(\frac{2 - 3y}{2}\right) = \frac{20}{1}\left(\frac{y + 51}{20}\right)$

$16 - 4y - 20 + 30y = y + 51$

$16 - 20 - 4y + 30y = y + 51$

$-4 + 26y = y + 51$

$26y - 1y = 51 + 4$

$25y = 55$

$y = \frac{55}{25}$

$y = 2\frac{1}{5}$

END OF LESSON 8

LESSON RECORD FORM

0902 Math 9
Revised 11/90

FOR STUDENT USE ONLY

Date Lesson Submitted

_____

Time Spent on Lesson

_____

(If label is missing or incorrect)

File Number

_____

Lesson Number _____

FOR A.D.L.C. USE ONLY

Assigned Teacher: _____

Lesson Grading: _____

Additional Grading E/R/P Code: _____

Mark: _____

Graded by: _____

Assignment Code: _____

Date Lesson Received:

_____

Lesson Recorded _____

Student's Questions and Comments

Apply Lesson Label Here

Name

Address

Postal Code

Please verify that preprinted label is for correct course and lesson.

Teacher's Comments:

_____

Correspondence Teacher

St. Serv. 14-91

ALBERTA DISTANCE LEARNING CENTRE

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1. BEFORE MAILING YOUR LESSONS, PLEASE SEE THAT:

(1) All pages are numbered and in order, and no paper clips or staples are used.
(2) All exercises are completed. If not, explain why.
(3) Your work has been re-read to ensure accuracy in spelling and lesson details.
(4) The Lesson Record Form is filled out and the correct lesson label is attached.
(5) This mailing sheet is placed on the lesson.

2. POSTAGE REGULATIONS

Do **not** enclose letters with lessons.

Send all letters in a separate envelope.

3. POSTAGE RATES

First Class

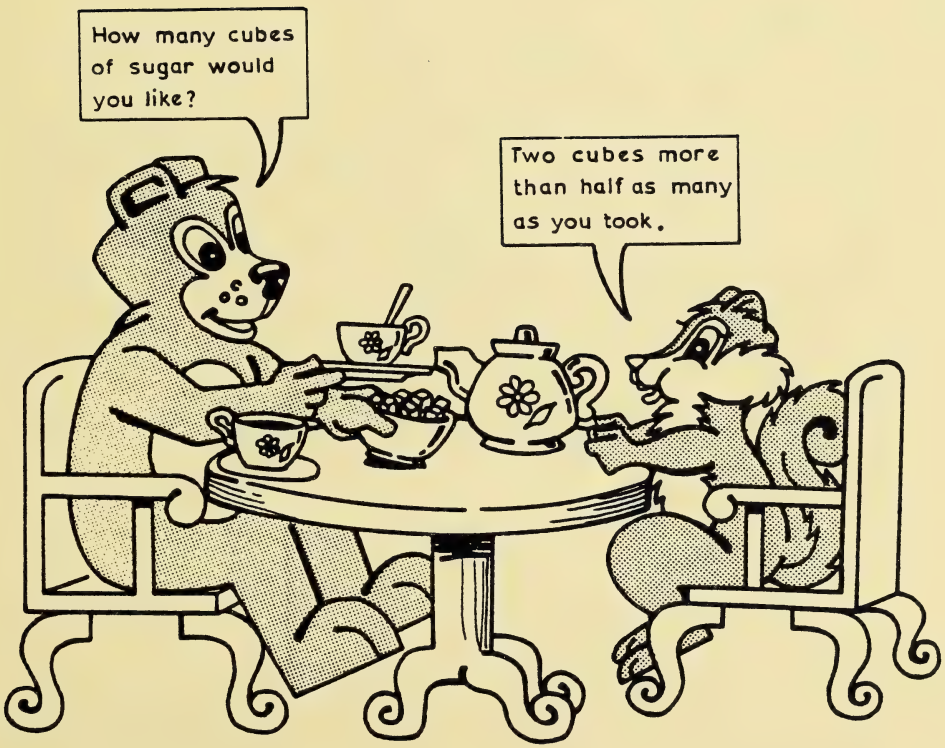
Take your lesson to the Post Office and have it weighed. Attach sufficient postage and a green first-class sticker to the front of the envelope, and seal the envelope. Correspondence lessons will travel faster if first-class postage is used.

Try to mail each lesson as soon as it has been completed.

When you register for correspondence courses, you are expected to send lessons for correction regularly. Avoid sending more than two or three lessons in one subject at the same time.

St. Serv. 9-91

PROBLEM SOLVING I



In this lesson you will learn about:

- Problem Solving
- Number Problems
- Measurement Problems

Keeping Skillful

Exercise 1

1. Use the distributive property to simplify the following.

(a) $5\ (3n + 2)$

(b) $6\ (x - 5)$

(c) $4\ (5n - 7)$

(d) $-2\ (3x + 1)$

(e) $-5\ (4x - 2)$

(f) $-1\ (-3x + 5)$

2. Solve the following equations:

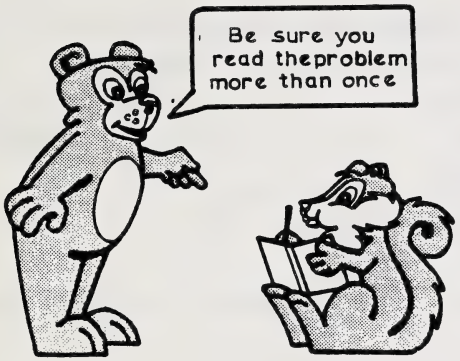
(a) $2x + 3 = 17$

(b) $5 - 3x = 14$

(c) $4\ (5x - 7) = 19x - 4$

(d) $3\ (2x - 3) = 2\ (x - 1)$

Reading a Word Problem



Much of your study of mathematics is concerned with solving word problems. Many students have difficulty with problem solving - not because they have trouble with the **mathematical ideas,** but because they do not understand the problem through their **reading.**

You cannot read a word problem in mathematics in the same way that you read a comic book or a short story. You cannot just skim quickly. It is important to read slowly and carefully. Moreover, you must read the problem more than once. Each reading should be done with a different purpose in mind. Below is a system for reading word problems.

System for Reading Word Problems

First Reading

Purpose: to find the main idea, or what the problem asks you to find out

Rate: fairly quickly

Second Reading

Purpose: to determine implied details or to figure out what the problem writer expects you to know. (For example, the perimeter of a rectangle is equal to twice the length plus twice the width.)

Rate: slowly

Third Reading:

Purpose: to locate the relevant details or the details needed to solve the problem

Rate: slowly and carefully

Reading usually will **not** tell you exactly which mathematical operation you need to perform on the numbers to arrive at your answer. This depends on your knowledge of mathematics. There are, however, certain words and phrases in the problem that help you to know which operation will solve the problem.

The following English words can be used to indicate the operations of addition, subtraction, multiplication, and division.

ADDITION	SUBTRACTION	MULTIPLICATION	DIVISION
sum plus more than increased by	difference less than minus decreased by diminished by	times product of doubled, tripled... twice, half...	the quotient of

Exercise 2

Which operation would you use if you read these phrases.

(a) how many altogether? _____

(b) how many are left? _____

(c) how many more? _____

(d) how many fewer? _____

(e) how many in all? _____

(f) how many at this rate? _____

(g) find the sum? _____

(h) find the product? _____

(i) find the difference? _____

(j) find the average? _____

Solving an Arithmetic Word Problem

Use the system for reading word problems to help you understand the problem. It may also help to make a sketch if possible.

Example:

What is the circumference of the largest circle than can be cut out of a square board with side 1.5 m?

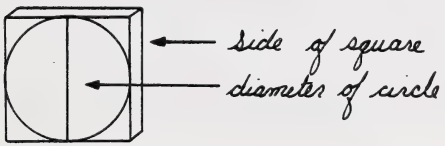
1. **Read to find the main idea:**

   What is the circumference of the largest circle that can be cut out?

2. **Read to find the implied details:**

   The circumference of a circle = πd.
   The diameter of the circle would equal the length of the side of the square.

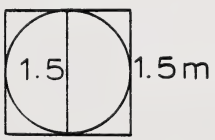
   This would be the biggest possible circle.



3. **Read to find the relevant details:**

   The length of the side of the square = 1.5 m. Therefore, the diameter of the circle is 1.5 m.

4. **Solution:**



$C = \pi d$
$C = 3.14 \times 1.5$
$C = 4.71$

The circumference is 4.71 m.

Self-Correcting Exercise A

Solve the following problem.

Mr. Thorn planted tomato plants in a rectangular space 4 metres long and 3 metres wide. If a box of plants could be planted in a space of 2 square metres, how many boxes of plants did Mr. Thorn have to buy?

1. **The main idea is:**

2. **The implied details are:**

3. **The relevant details are:**

4. **Solution**

Check your answers with those on page 25 of this lesson.
Correct any errors in your work.

EXERCISE 3

Solve the following problems.

(a) A market gardener has 16 rows of tomato plants with 36 plants in each row. If the crop of tomato was 2880 kg, what was the average yield per plant?

1. **The main idea is:**

2. **The implied details are:**

3. **The relevant details are:**

4. **Solution:**

(b) The Caribou gold strike in 1861 helped open the interior of British Columbia for development. How long will it be until the bicentennial of the gold strike?

1. **The main idea is:**

2. **The implied details are:**

3. **The relevant details are:**

4. Solution:

(c) A master bedroom is 6 m long and 5 m wide. Find the cost of carpeting the room with rug at \$9.75 per m^2.

1. **The main idea is:**

2. **The implied details are:**

3. **The relevant details are:**

4. **Solution:**

(d) In an archery competition, Joseph shot 36 arrows at a small target 30 m away. If he retrieved the arrows from the target each time he had shot 6 arrows, how far did he walk?

1. **The main idea is:**

2. **The implied details are:**

3. **The relevant details are:**

4. Solution:

(e) Mary makes $2.00 an hour babysitting. She is saving her money to buy Christmas presents for her family. She plans to buy her father a pair of gloves for $14.95, her mother some perfume for $8.75, her sister a book for $3.95 and her brother a puzzle for $2.35. How many hours will she have to babysit to earn the money for the gifts?

1. **The main idea is:**

2. **The implied details are:**

3. **The relevant details are:**

4. Solution:

Using Algebra to Solve Problems

While some problems can be solved using only arithmetic, many problems require a knowledge of algebra. In this lesson and the next lesson you will learn an organized method to solve these types of word problems.

Below are the steps in the problem solving method.

Problem Solving Method

1. Read to find the main idea.
2. Read to find the implied details.
3. Read to find the relevant details.
4. Identify the unknown and represent the unknown by a variable. (If there are more than one unknowns, represent the other unknowns by relating them to this variable).
5. Write an equation.
6. Solve the equation.
7. Write a statement answering the question in the problem.
8. Check your answer.

Number Problems

Example 1

Three times a certain whole number is eighteen more than the number itself. Find the number.

1. **Main idea.**

   You are to ***find the number.*** Let the unknown number be n.

2. **Implied details.**

   In this problem there are no implied details.

3. **Relevant details.**

   $\underset{3}{\textit{Three}}\ \underset{\times}{\textit{times}}\ \textit{a certain}\ \underset{n}{\textit{whole number}}\ \underset{=}{\textit{is}}$
   $\underset{18}{\textit{eighteen}}\ \textit{more}\ \underset{+}{\textit{than}}\ \textit{the number}\ \underset{n}{\textit{itself.}}$

4. **Variable.**

Let n be the number.

5. **Equation.**

$3 \times n = 18 + n$

6. **Solution.**

$3n + -n = 18 + n + -n$

$2n = 18$

$n = 9$

7. **Statement.**

The number is 9.

8. **Check**

Does $3 \times 9 = 18 + 9$? Yes.

Example 2

The sum of three consecutive numbers is eighty-four. Find the numbers.

1. **Main idea.**

You are to find the three numbers.

2. **Implied details.**

Sum means to add.

Consecutive means that the numbers follow one another. For example 5, 6, 7 are consecutive numbers. (6 is one more than 5. 7 is one more than 6.)

So, let the lowest number be n

the next number be $n + 1$

the largest number be $n + 1 = n + 2$

3. **Relevant details.**

$n + (n + 1) + (n + 2) = 84$

The *sum* of three *consecutive* numbers is eighty-four.

4. **Variable.**

Let the numbers be n, n + 1 and n + 2

5. **Equation.**

$n + (n + 1) + (n + 2) = 84$

6. **Solution.**

$n + n + 1 + n + 2 = 84$

$(n + n + n) + (1 + 2) = 84$

$3n + 3 = 84$

$3n + 3 + -3 = 84 + -3$

$3n = 81$

$\frac{3n}{3} = \frac{81}{3}$

$n = 27$

$n + 1 = 28$

$n + 2 = 29$

7. **Statement**

The numbers are 27, 28, 29.

8. **Check**

Does $27 + 28 + 29 = 84$? Yes

Example 3

Two numbers differ by twelve. If their sum is one hundred thirty-two, find the numbers.

1. **Main idea.**

You are to find two numbers.

2. **Implied details.**

"Two numbers differ by twelve" means that one number is twelve larger than the other.

Let the smaller number be n

Let the larger number be n + 12

"Sum" means add

3. **Relevant details.**

$n + (n + 12) = 132$

... their *sum* is one hundred thirty-two

4. **Variable.**

Let the numbers be n and n + 12

5. **Equation.**

$n + (n + 12) = 132$

6. **Solution.**

$n + n + 12 = 132$

$2n + 12 = 132$

$2n + 12 + (-12) = 132 + (-12)$

$2n = 120$

$\frac{2n}{2} = \frac{120}{2}$

$n = 60$

$n + 12 = 72$

7. **Statement.**

The numbers are 60 and 72.

8. **Check**

Does 60 + 72 = 132? Yes.

SELF-CORRECTING EXERCISE B

Write equations only for the following word problems. Let the numbers be n.

1. Seven added to three times a number is equal to forty-three. Find the number.

_____

2. Five times a number, decreased by eight is equal to twenty-seven. Find the number.

_____

3. Two numbers differ by seven. If the sum of the numbers is sixty-five, find the numbers.

_____

4. Find three consecutive numbers whose sum is 75.

_____

Check your answers with those on page 25 of this lesson. Correct any errors in your work.

Exercise 4

Write equations only for the following word problems.

1. Four times a certain number diminished by two is equal to forty-eight increased by two times the number. Find the number.

_____

2. When a certain whole number is doubled, the result is twenty-three more than the number itself. Find the number.

_____

3. Find the number which when divided by four and increased by twelve is the same as when it is divided by three and decreased by five.

_____

4. One number is six more than another number. If three times the smaller number minus the larger number is twenty-three what are the numbers?

_____

5. Find two consecutive numbers so that three times the larger less two times the smaller is forty-three.

_____

Exercise 5

Using the problem solving method, solve the following problems. The first three steps can be done mentally. Show the final five steps.

1. If tripling a certain number produces the same result as increasing the number's double by four. Find the number.

Variable: Let x be the number

Equation: $3x = 2x + 4$

Solution: $3x - 2x = 2x + 4 - 2x$

$x = 4$

Statement: The number is 4

Check: Does $3 \times 4 = 2 \times 4 + 4$?

Yes

2. Multiplying a certain whole number by seven has the same result as increasing twice the original number by twenty-five. Find the number.

(3) Three times a certain whole number minus thirteen is the same amount as eleven minus the number itself. Find the number.

(4) One number is twelve more than another. When the larger is divided by the smaller, the answer obtained is four. Find the numbers.

(5) Two numbers differ by five. One half of the larger number is the same as eight added to one-third of the smaller number. Find the numbers.

(6) The sum of three consecutive numbers is ninety-six. Find the numbers.

Measurement Problems

With measurement problems it is helpful to make a sketch.

Example 1

The length of a rectangle is 6 cm more than its width. If the perimeter is 132 cm, find the dimensions of the rectangle.

1. **Main idea.**

   You are to find the dimensions of the rectangle.

2. **Implied details.**

   (a) *"dimensions"* means the length and the width of the rectangle.

   (b) *"perimeter"* is the distance around the figure. For a rectangle P = 21 + 2w.

3. **Relevant details.**

   (a) *The length of the rectangle is 6 cm more than its width.*

   Let the width be w. Then the length would be w + 6.

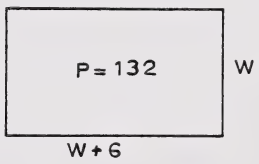
   (b) *The perimeter is 132 cm.*

4. **Variable.**

   Let w be the width and w + 6 be the length.

5. **Equation.**

   P = 21 + 2w



6. **Solution.**

   $132 = 2(w + 6) + 2(w)$

   $132 = 2w + 12 + 2w$

   $132 = 2w + 2w + 12$

   $132 = 4w + 12$

   $132 + (-12) = 4w + 12 + (-12)$

   $120 = 4w$

   $\frac{120}{4} = \frac{4w}{4}$

   $30 = w$

   $w = 30$ cm

   $w + 6 = 36$ cm

7. **Statement.**

The length is 36 cm and the width is 30 cm.

8. **Check.**

$P = 21 + 2w = 2(36) + 2(30) = 72 + 60 = 132$

Example 2

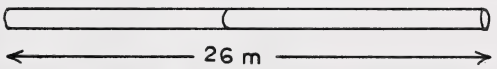
A 26 m garden hose is made up of two separate hoses. The longer hose is 4 m shorter than twice the length of the other. Find the length of the two hoses.

1. **Main idea.**

You are to find the length of the two hoses.

2. **Implied details.**

The hoses when added together are 26 m long.



3. **Relevant details.**

The longer hose is ***4 m shorter than twice the length of the other.***

"Shorter" means subtract. Watch the order. "Twice" means "2 times".

4. **Variable.**

Let x be the shorter hose and $2x - 4$ be the longer hose.

5. **Equation.**

$x + (2x - 4) = 26$

6. **Solution.**

$x + 2x + (-4) = 26$

$3x - 4 = 26$

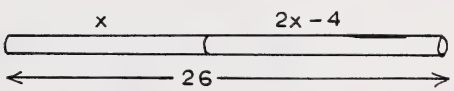
$3x - 4 + 4 = 26 + 4$

$3x = 30$

$\frac{3x}{3} = \frac{30}{3}$

$x = 10$

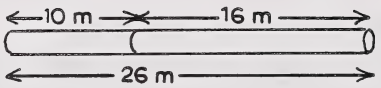
$2x - 4 = 2(10) - 4 = 20 - 4 = 16$



7. **Statement.**

The hoses are 16 m and 10 m long.

8. **Check.**



$$\begin{array}{r} 10 \\ +\ 16 \\ \hline 26 \end{array}$$

Self-Correcting Exericse C

Write variable expressions for the length of each of the following rectangles and place on the sketch. Let the width be W.

(a) The length is double the width.

(b) The difference between the length and the width is 9 m.

(c) The length is three times the width.

(d) The length of a rectangle is 6 cm more than the width.

(e) The length of a rectangle is 5 more than twice the width.

Check your answers with those on page 26 of this lesson. Correct any errors in your work.

Exercise 6

Write variable expressions for the dimensions of the following rectangles. Place these on the diagrams.

(a) The length is twice the width.

(b) The length is 2 m longer than twice the width.

(c) The length is 3 cm shorter than twice the width.

(d) The length is 4 cm longer than four times the width.

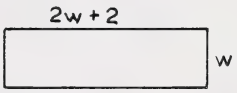
(e) The length and width differ by 12.

Exercise 7

Solve the following problems. Make a diagram for each. Perform all the steps in the problem solving method. The first three steps can be done mentally but show the last five steps.

1. The length of a room is 2 m more than twice the width. Find the dimensions of the room, if the perimeter of the room is 34 m.

Variable: Let the width be w and the length be $2w+2$.



Equation: $2(2w+2)+2w=34$

Solution:
$$4w+4+2w=34$$
$$4w+2w+4=34$$
$$6w+4=34$$
$$6w+4-4=34-4$$
$$6w=30$$
$$\frac{6w}{6}=\frac{30}{6}$$
$$w=5$$
$$2w+2=2(5)+2=10+2=12$$

Statement: The length is 12 m and the width is 5 m.

Check: Does $2(12)+2(5)=34$? Yes.

2. An irrigation pond is enclosed by a fence. The length is 30 m more than 6 times the width. If the perimeter is 172, find the dimensions of the fence.

3. On a certain power line hydro towers are built the same distance apart. If a 500 m cable stretches tightly along five towers, what is the distance between each tower?

4. A 500 cm piece of lumber is cut into two pieces. If one piece is eight more than one half the other, how big are the pieces?

Key to Self-Correcting Exercises

Self-Correcting Exercise A, page 6

1. **The main idea is.**

   how many boxes of plants did Mr. Thorn have to buy?

2. **The implied details are.**

   (a) The area of a rectangle is found by multiplying the length by the width. $A = l \times w$

   (b) The number of boxes is found by dividing the total area by the area each box will cover.

3. **The relevant details are.**

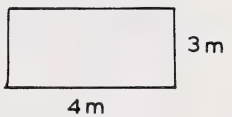
   (a) The garden is a 4 m by 3 m rectangle.

   (b) A box of plants can be planted in an area 2 m^2.

4. Solution.

   $A = l \times w$

   $A = 3 \times 4 = 12\ m^2$

   The number of boxes $= \frac{12}{2} = 6$

   6 boxes of plants were needed.

Self-Correcting Exercise B, pages 15 and 16

1. $3n + 7 = 43$

2. $5n - 8 = 27$

3. $n + (n + 7) = 65$ if n = the smaller number.

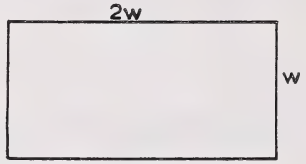
   or

   $n + (n - 7) = 65$ if n = the larger number.

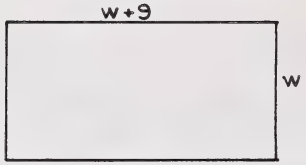
4. $n + (n + 1) + (n + 2) = 75$

Self-Correcting Exercise C, page 22

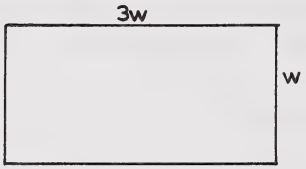
a)



b)



c)



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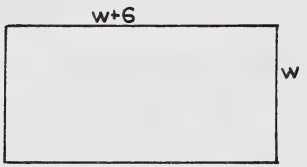

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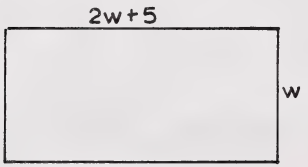

End of Lesson 9

LESSON RECORD FORM

0902 Math 9
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Date Lesson Submitted

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File Number

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REVIEW

This lesson will review the concepts taught in Lessons 1 to 9.

PART A

Lesson 1

1. Show the magnitude and the direction for the following questions. (4 marks)

   (a) 500 m below sea level _____

   (b) an incline of 30 cm _____

   (c) a deficit of $10 _____

   (d) 45 minutes ahead of schedule _____

Lesson 1

2. Do the following additions and subtractions. (9 marks)

   (a) 15 - (-9) =

   (b) 23 + (-3) =

   (c) -7 - (+10) =

   (d) -33 + 29 =

   (e) -9 - (-9) + 9 =

   (f) -8 + 15 + 16 =

   (g) 24 - 9 - 4 =

   (h) 25 + (-31) - 29 + 5 =

   (i) 106 + 55 - 152 - 4 =

Lesson 1

3. Tell only whether the answers to these questions will be positive or negative. (5 marks)

(a) $4 + 5 - 10 =$ _____

(b) $-15 - 14 - (+40) =$ _____

(c) $-4 \times 16 \times -2 =$ _____

(d) $25 \div -5 \times -7 =$ _____

(e) $-75 \times (-100) \div -2 =$ _____

Lesson 1

4. Review the rules for order of operations before doing these questions. Show all steps. (10 marks)

(a) $-19 - (8 + (-6)) =$

(b) $9 \times -7 + 4 =$

(c) $24 \div 8 - 5 =$

(d) $32 + 8 \times 8 - 4 =$

4. (e) $\dfrac{24 \div (10 - 2) - (-4)}{35 \div -5 + (-14)} =$

Lesson 2

5. **Evaluate the following.** Show all steps. (12 marks)

(a) $5^4 =$

(b) $10^0 =$

(c) $(-3)^2 =$

(d) $(-3)^{-4} =$

(e) $-3^4 =$

(f) $4^{-2} =$

Lesson 2

6. Evaluate the following. Show all steps. (12 marks)

(a) $3(5)^2 =$

(b) $[2(4)]^2 =$

(c) $9^3 \div 9^2 =$

(d) $2^3 \times 2^2 =$

(e) $(3^2)^3 =$

(f) $[(-3)^2]^4 =$

Total for Part A _____ (maximum possible: 52 marks)

PART B

Lesson 3

1. Find the G.C.F. of 54 and 60. (2 marks)

Lesson 3

2. Find the L.C.M. of 18 and 24. (2 marks)

Lesson 3

3. Perform the indicated operations. Show your work. (8 marks)

(a) $2\frac{3}{4} + 1\frac{2}{3} =$

(b) $5\frac{1}{6} - 2\frac{2}{3} =$

(c) $-4 - 2\frac{2}{5} =$

(d) $-8\frac{1}{2} + 1\frac{11}{12} =$

Lesson 4

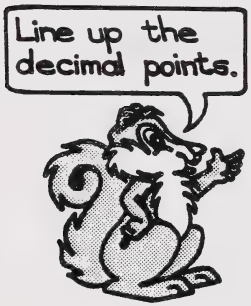
4. **Perform the indicated operations.** Show your work. (12 marks)

(a) $9\frac{3}{5} \times 4\frac{3}{8} =$

(b) $10\frac{2}{3} \div \frac{1}{6} =$

(c) $88 + 12.03 + 0.094 =$

(d) $9 - 7.3 =$



(e) $2.56 \times 0.03 =$

(f) $9.6 \times 0.32 =$

Lesson 4

5. Round off the following to the nearer hundredth. (4 marks)

(a) 45.061 8 _____

(b) 2.999 _____

Lesson 5

6. Solve the following. Show your solution steps. (10 marks)

(a) $7 = c + 24$

(b) $9 - x = 32$

(c) $3m - 7 = 11$

(d) $\frac{8}{9}x = 16$

(e) $9y - 14 = 6 + 7y$

Lesson 5

7. Write the following in mathematical form. (2 marks)

(a) 6 less than twice a number. _____

(b) the square of a number increased by 17. _____

Lesson 8

8. Solve the following. Show all steps. (8 marks)

(a) $\frac{x + 7}{5} = \frac{10 - x}{7}$

(b) $\frac{x}{5} + \frac{2x + 1}{3} = \frac{4 - x}{2}$

Total for Part B _____ (maximum possible: 48 marks)

PART C

Lesson 6

1. Write the following decimals in the form of a fraction: Show all your work. (6 marks)

(a) $8.\dot{3}\dot{4}$

(b) $5.7\dot{9}$

Lesson 6

2. Write the following numbers in scientific notation. (5 marks)

(a) 0.00069 _____

(b) 429 _____

(c) 5000 _____

(d) 0.01 _____

(e) 147.6 _____

3. Perform the indicated operations. (7 marks)

(a) 64×10^{-5} = _____

(b) $0.007 \div 10^{3}$ = _____

(c) $33.03 \div 10^{4}$ = _____

(d) 7.63×10^{6} = _____

(e) 8×10^{3} = _____

(f) 5.003×10^{-3} = _____

(g) $0.9 \div 10^{-6}$ = _____

Lesson 9

4. Ten more than a certain whole number is twenty less than twice the number. Find the number. Show your work. (5 marks)

5. A rectangular field with length 8 more than 3 times its width has a perimeter of 2016 m. Find the width of the field. Show all steps. (5 marks)

Lesson 7

6. Change these percents to decimal form. (4 marks)

(a) 77%

(b) 147%

(c) $\frac{1}{2}$%

(d) 1.75%

Lesson 7

7. Change the following fractions and mixed numbers to a percent. (8 marks)

(a) $\frac{1}{4}$

(b) $\frac{7}{8}$

(c) $1\frac{4}{5}$

(d) $7\frac{3}{4}$

Lesson 7

8. For each of the following, put the problem into the general sentence form "a is r% of b". (4 marks)

(a) 62 is what percent of 74?

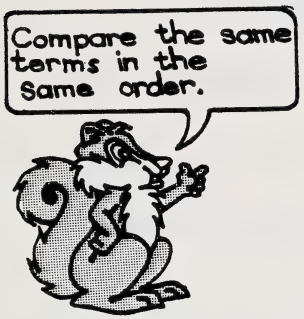
_____

(b) 82% of what number is 46?

_____

Lesson 7

9. Given the following proportions, find the unknown. Show all steps. (6 marks)



(a) $8 : 5 = 4 : x$

(b) $2 : k : 7 = p : 40 : 56$

Total for Part C _____ (maximum possible: 50 marks)

Total for Review _____ (maximum possible 150 marks)

END OF LESSON 10

LESSON RECORD FORM

0902 Math 9
Revised 11/90

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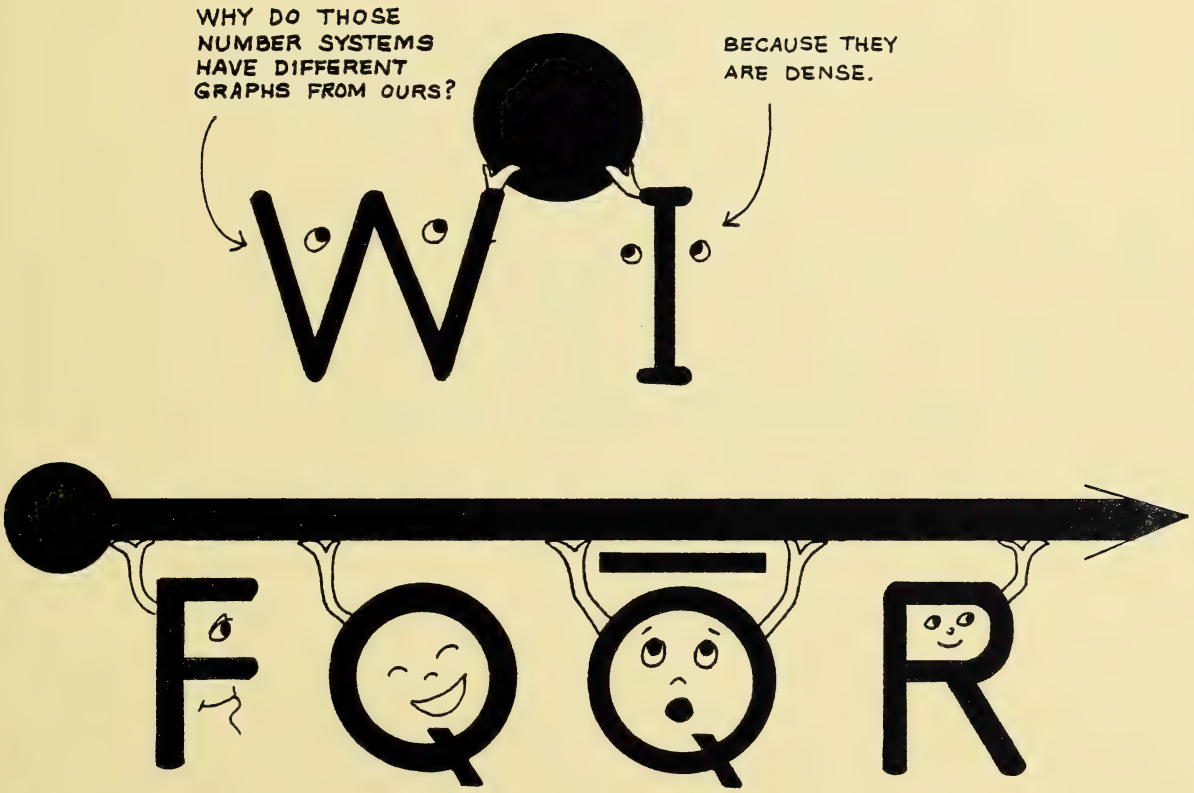
St. Serv. 9-91

Number Systems and their Graphs



In this lesson you will learn about:

- graphing whole numbers, integers, rational numbers, irrational numbers and real numbers
- finding square roots

KEEPING SKILLFUL

EXERCISE 1

1. Evaluate the following.

(a) $5^4 =$

(b) $(-3)^5 =$

(c) $8^0 =$

(d) $7^{-2} =$

2. Solve for the unknown in the following ratios.

(a) $\frac{9}{33} = \frac{6}{x}$

(b) $\frac{y}{21} = \frac{42}{3}$

GRAPHING WHOLE NUMBERS AND INTEGERS

Reference Read pages 2 and 35 of *Holt Mathematics 3.*

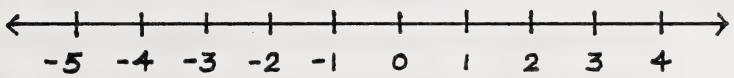
The set of whole numbers is:

$$W = \{0, 1, 2, 3, \ldots\}$$

The set of integers is:

$$I = \{\ldots -3, -2, -1, 0, 1, 2, 3, \ldots\}$$

Recall that a number line is shown like this:



Notice that the positive numbers are on the right and the negative numbers are on the left. Since it is a line, there are arrows at both ends.

To graph a solution set from the replacement set of whole numbers or integers, you mark each of the solutions on a number line using a heavy dot.

Example 1 Graph the solution of $2x + 3 = 11$, where $x \in W$.

Note: The symbol $\in$ (epsilon) means "is a member of". Therefore, $x \in W$ means that the solution x must be a member of the set of whole numbers. In this case the set of whole numbers is the replacement set for x.

$$2x + 3 = 11$$

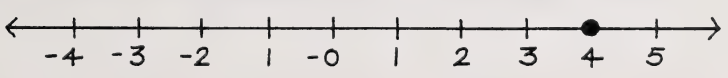
$$2x = 11 - 3$$

$$2x = 8$$

$$x = 4$$

The solution is {4}.

The solution, 4, is marked on the number line.



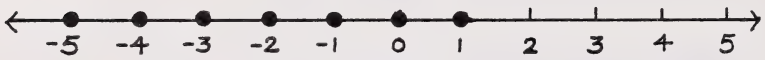
For several years you have probably been using these signs:

$>$	greater than
$\geq$	greater than or equal to
$<$	less than
$\leq$	less than or equal to

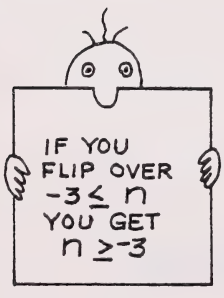
Example 2 Graph the solution set of $x \leq 1$, $x \in I$.

The solution set contains all integers less than or equal to 1. The solution set is $\{. \ . \ . \ -2, -1, 0, 1\}$. This set is infinite because the possible solutions go on forever.

To show on the graph that the solutions continue on to the left to infinity, mark all the members up to the arrow on the number line.



You know that $2 < 3$ means the same as $3 > 2$. Thus, $x \leq 1$ is the same as $1 \geq x$. Similarly, $9 > n$ is the same as $n < 9$.



Consider $-3 \leq n < 5$, $n \in I$. This means $-3 \leq n$ and $n < 5$. This is the same as $n \geq -3$ and $n < 5$.

Which integers fit the condition $n \geq -3$ and also the condition $n < 5$? The solution set is $\{-3, -2, -1, 0, 1, 2, 3, 4\}$.

Example 3 Graph the solution set of $-3 \leq n < 5$, $n \in I$.

In the explanation on page 4 it was established that the solution set is {-3, -2, -1, 0, 1, 2, 3, 4}. Notice that this set is finite.

In this case it is important that some unmarked numbers are shown on the left and right-hand sides of the graph. Why? Marking the numbers up to the arrow would have indicated an infinite set.

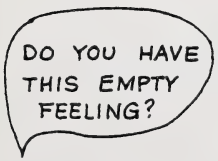
Example 4 Graph the solution set of $-5 < n < 0$, $n \in W$.

$-5 < n < 0$ means that $-5 < n$ and $n < 0$.

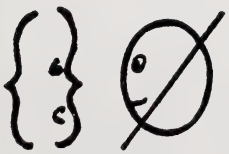
This means that $n > -5$ and $n < 0$.

Which whole numbers are greater than -5 and less than 0? You may think of the numbers -4, -3, -2 and -1, but these are <u>not</u> whole numbers.

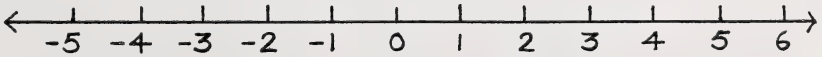
Since no whole numbers fulfill the conditions $n > -5$ and $n < 0$, the solution set is the empty set.



The empty set can be represented by an empty set of brackets { } or the symbol $\emptyset$ (named phi).



The graph would be a number line with no numbers marked.



SELF-CORRECTING EXERCISE A

To practice finding solution sets do the following exercises. Do not draw the graphs.

1. $5n + 2 = 17$

2. $n > 5$

3. $3 < n \quad 8$

4. $20 \quad n < 21$

5. $10 > n \quad 3$

6. $12 \quad n \quad 10$

7. $0 < n \quad 5$

8. $n \quad 6$

9. $8 < n < 9$

10. $3n = 0$

Empty Sets _____

Infinite Sets _____

Check your answers with those on page 21 of this lesson.

EXERCISE 2 Write the solution set for each of the following. Then, draw each graph.

1. $4n + 14 = 22 \qquad (n \in W)$

2. $y \leq -2 \qquad (y \in I)$

3. $4 > x \qquad (x \in W)$

4. $-4 \leq a < 3 \qquad (a \in I)$

FRACTIONS AND RATIONAL NUMBERS

Reference

Study column one on page 6 and column one on page 66 of *Holt Mathematics 3.*

The set of fractions is:

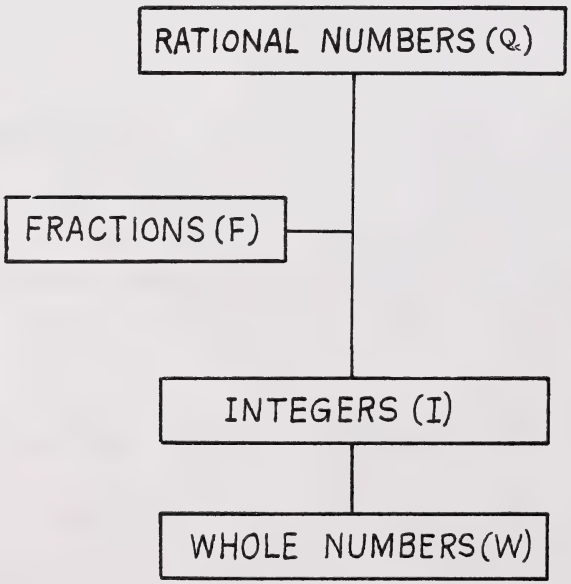
$$F = \{ \frac{a}{b} \ / \ a, \ b \in W, \ b \neq 0 \}$$

Whole numbers and mixed numbers can be written in the form of a fraction, $\frac{a}{b}$. Thus, they are members of the set of fractions.

The set of rational numbers is:

$$Q = \{ \frac{a}{b} \ / \ a, \ b \in I, \ b \neq 0 \}$$

Notice that the set of fractions is defined to be positive, while rational numbers can be either positive or negative. The set of rational numbers includes whole numbers, integers and fractions.

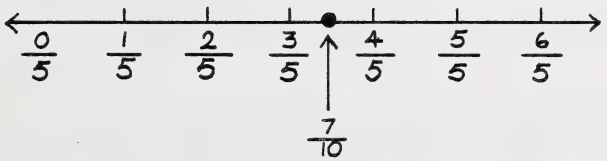


Density Fractions and rational numbers are said to be dense.

Example 1 Choose two fractions, such as $\frac{3}{5}$ and $\frac{4}{5}$. Is there another fraction between these two fractions?

If you add $\frac{3}{5}$ and $\frac{4}{5}$ and divide by two, you will find another fraction midway between them.

$$\frac{\frac{3}{5}+\frac{4}{5}}{2} = \frac{\frac{7}{5}}{2} = \frac{7}{5} \times \frac{1}{2} = \frac{7}{10}$$

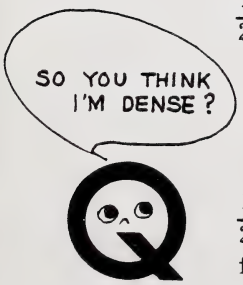


$\frac{7}{10}$ is midway between $\frac{3}{5}$ and $\frac{4}{5}$.

Example 2 Find the fraction midway between $\frac{3}{5}$ and $\frac{7}{10}$.

$$\frac{\frac{3}{5}+\frac{7}{10}}{2} = \frac{\frac{6}{10}+\frac{7}{10}}{2} = \frac{\frac{13}{10}}{2} = \frac{13}{10} \times \frac{1}{2} = \frac{13}{20}$$

$\frac{13}{20}$ is midway between $\frac{3}{5}$ and $\frac{7}{10}$. Mark it on the graph above.



You could now find the fraction midway between $\frac{3}{5}$ and $\frac{13}{20}$, and so on. Between any two fractions there is another fraction. The same is true for rational numbers. Thus, fractions and rational numbers are said to be dense.

EXERCISE 3 1. Find the fraction midway between the given fractions.

(a) $\frac{4}{9}$ and $\frac{5}{9}$ (b) $\frac{1}{5}$ and $\frac{1}{6}$

2. Find the rational number midway between the following rational numbers.

(a) $-\frac{5}{6}$ and $-\frac{5}{7}$ (b) $-2\frac{1}{3}$ and $-2\frac{2}{3}$

GRAPHING FRACTIONS AND RATIONAL NUMBERS

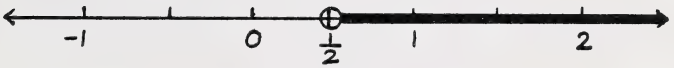
How would you graph $x > \frac{1}{2}$, $x \in F$? You can probably think of many fractions greater than $\frac{1}{2}$, such as $\frac{3}{5}$, $\frac{9}{11}$, $\frac{3}{2}$, $\frac{20}{6}$, $\frac{100}{4}$, and so on. How can you be sure to list all of them? Quite simply, you cannot. Between any two fractions you can always find another fraction.

To graph whole numbers and integers you mark with a heavy dot each of the members from the solution set. However, if you mark all of the fractions greater than $\frac{1}{2}$ this way, the dots will overlap.

Instead of using heavy dots, it is easier to draw a solid line.

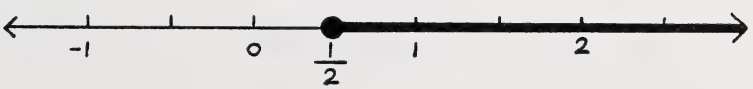
Thus, both fractions and rational numbers are graphed by a solid line.

Here is the graph of $x > \frac{1}{2}$, $x \in F$.



Notice that the graph started with an *open circle*, which indicates that 1/2 is not part of the solution.

Here is the graph of $x \geq \frac{1}{2}$, $x \in F$

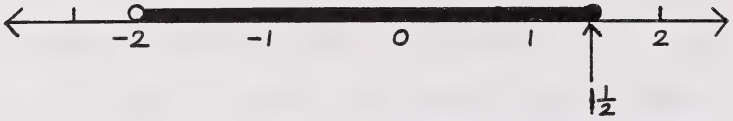


This graph started with a *closed circle* or heavy dot, which indicates that 1/2 is part of the solution.

To show that the graphs of $x > \frac{1}{2}$ and $x \geq \frac{1}{2}$ are infinite, the solid line is continued up to the arrow.

Example Graph $-2 < x \leq 1\frac{1}{2}$, $x \in Q$

This is the same as $x > -2$ and $x \leq 1\frac{1}{2}$.



○ *means that the number is not in the solution set.*

● *means that the number is in the solution set.*

Notice that the equals sign indicates whether you should use an open or closed door.

SELF-CORRECTING EXERCISE B

Graph the solution set for each of the following.

1. $x > -9$, $x \in Q$.

2. $x \leq 3\frac{1}{2}$, $x \in F$.

3. $-4.5 \leq x < 0.5$, $x \in Q$.

Check your answers with those on page 21 of this lesson. Correct any errors in your work.

EXERCISE 4 Graph the solution set for each of the following.

1. $-2\frac{3}{4} < x,\ x \in Q.$

2. $y \leq 3.75,\ y \in Q.$

3. $2\frac{1}{2} < a < 9,\ a \in F.$

4. $-3\frac{3}{4} \leq c < 3,\ c \in Q.$

Rational Numbers

A rational number is any number that can be written in the form of a fraction.

In Lesson 6, you learned that a fraction can be changed to either a terminating or repeating decimal.

Therefore, a rational number in decimal form either terminates or repeats.

Are all numbers rational numbers? No. The number 0.131131131113... neither terminates or repeats, so it is not rational.

Irrational Numbers

A decimal number that is not rational is irrational. An irrational number is a non-terminating, non-repeating decimal. Some examples are:

0.12345678910111213 . . .

0.172737475767 . . .

1.22333444455555 . . .

$\overline{Q}$ is the symbol for the set of irrational numbers.

SELF-CORRECTING EXERCISE C

Indicate whether the number is rational or irrational.

1. (a) -1.245 (b) $0.\dot{3}\dot{4}$ (c) $\sqrt{3}$

(d) π (e) $\sqrt{25}$ (f) $2\sqrt{6}$

Check your answers with those on page 21 of this lesson.

REAL NUMBERS

Reference Read page 102 of *Holt Mathematics 3*.

Definition The set of real numbers (R) is the union of the set of rational numbers (Q) and the set of irrational numbers ($\overline{Q}$).

$$R = Q + \overline{Q}$$

For this level of mathematics, you can think of real numbers as the set of "all" numbers.

On page 9 of this lesson you learned that between any two fractions you can always find another fraction. Thus, fractions are said to be dense. This also holds true for rational numbers, irrational numbers and real numbers. To graph sets of these numbers, use solid lines.

Summary

$W = \{0, 1, 2, 3 \ldots\}$

$I = \{\ldots -2, -1, 0, 1, 2 \ldots\}$

These sets are not dense. To graph these sets, individually mark each of the numbers.

$F = \{\frac{a}{b} / a, b \in W, b \neq 0\}$

$Q = \{\frac{a}{b} / a, b \in I, b \neq 0\}$

$\overline{Q}$ = any number that is not rational. They can be expressed as non-terminating, non-repeating decimals.

These sets are dense. To graph these sets, use solid lines.

$R = Q + \overline{Q}$

Exercise 5 Answer true or false.

1. R is the symbol for the set of rational numbers. _____

2. $\overline{Q}$ is the symbol for the set of rational numbers. _____

3. Fractions can be positive or negative. _____

4. Whole numbers can be positive or negative. _____

5. Rationals can be positive or negative. _____

6. Fractions are a dense set. _____

7. Whole numbers are a dense set. _____

8. Integers are a dense set. _____

9. Rationals are a dense set. _____

10. To graph a dense set you individually mark each number. _____

Exercise 6

Graph the following sets. Be aware of the various replacement sets that have been used.

1. $\{x < 9, x \in W\}$

EXERCISE 6 (continued)

Graph the following sets.

2. $\{ -3 < y \leq 5,\ y \in I \}$

3. $\{ 2\frac{1}{4} > c,\ c \in F \}$

4. $\{ -7 \geq x > -9,\ x \in Q \}$

5. $\{ -3.75 \leq a \leq 0,\ a \in R \}$

SQUARE ROOTS

In Lesson 2 you learned that squaring a number means to multiply it by itself. For example:

$$x^2 = x \times x$$

$$7^2 = 7 \times 7$$

$$(-5)^2 = (-5) \times (-5)$$

To find the square root of a number, you must find the factor that, when multiplied by itself, equals the given number. $\sqrt{\ }$ is the symbol for square root.

$\sqrt{64} = 8$ because $8 \times 8 = 64$

$\sqrt{144} = 12$ because $12 \times 12 = 144$

Principal Square Root

When $\sqrt{\ }$ is written by itself you are being asked to find the positive square root, sometimes called the principal square root.

However, a negative square root does exist. This is shown by $-\sqrt{\ }$.

$-\sqrt{64} = -8$ because $-8 \times -8 = 64$

$-\sqrt{144} = -12$ because $-12 \times -12 = 144$

Sometimes you are asked to give both square roots. This is indicated by $\pm\sqrt{\ }$.

$\pm\sqrt{64} = \pm 8$ $\pm\sqrt{144} = \pm 12$

Reference

Read page 83 of *Holt Mathematics 3.*

Opposite Operations

Notice that squaring and finding the square root are opposite operations.

$\sqrt{4^2} = \sqrt{16}$ $\sqrt{16} = 4$

Therefore $\sqrt{4^2} = 4$

Opposite operations cancel each other out.

EXERCISE 7 1. Find the principal square root of each of the following.

(a) $\sqrt{100}$ = _____ (b) $\sqrt{121}$ = _____

(c) $\sqrt{169}$ = _____ (d) $\sqrt{400}$ = _____

(e) $\sqrt{1}$ = _____ (f) $\sqrt{225}$ = _____

2. Do the following.

(a) $-\sqrt{81}$ = _____ (b) $\pm\sqrt{441}$ = _____

(c) $\sqrt{15^2}$ = _____ (d) $\pm\sqrt{8^2}$ = _____

(e) $\sqrt{34^2}$ = _____ (f) $\pm\sqrt{12^2}$ = _____

3. Can you find $\sqrt{-9}$? _____

Explain your answer. _____

_____

_____

FINDING SQUARE ROOTS OF POWERS

In Lesson 2 you learned that $(a^x)^y = a^{xy}$.

Therefore $(x^4)^2 = x^{4 \times 2} = x^8$

or $(y^9)^2 = y^{9 \times 2} = y^{18}$

Thus, squaring a power is the same as multiplying the exponent by 2. It is therefore logical that taking the square root of a power is the same as dividing the exponent by 2.

$\sqrt{x^8} = x^4$ because, $x^4 \cdot x^4 = (x^4)^2 = x^8$

$\sqrt{9x^{20}} = 3x^{10}$ because, $3x^{10} \cdot 3x^{10} = (3x^{10})^2 = 9x^{20}$

EXERCISE 8 Find the square root of the following powers.

(a) $\sqrt{y^{14}} =$ _____ (b) $\sqrt{x^{30}} =$ _____

(c) $\sqrt{16x^6} =$ _____ (d) $\sqrt{25y^{36}} =$ _____

(e) $\sqrt{144n^8} =$ _____ (f) $\sqrt{81a^{26}} =$ _____

KEY TO SELF-CORRECTING EXERCISES IN LESSON 12

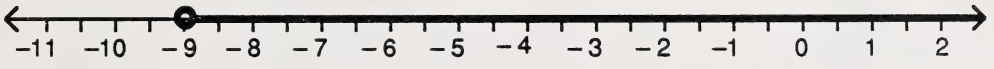
SELF-CORRECTING EXERCISE A, Page 6.

1. {3}	2. {6,7,8,...}	3. {4,5,6,7,8}
4. {20}	5. {3,4,5,6,7,8,9}	6. {10,11,12}
7. {1,2,3,4,5}	8. {6,7,8,...}	9. { }
10. {0}		

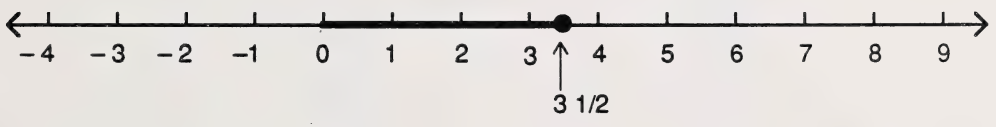
Empty Sets: 9
Infinite Sets: 2, 8

SELF-CORRECTING EXERCISE B, Page 12.

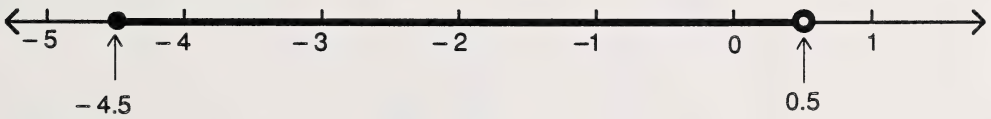
1. $x > -9$, $x \in Q$



2. $x \leq 3\frac{1}{2}$, $x \in F$



3. $-4.5 \leq x < 0.5$, $x \in Q$



SELF-CORRECTING EXERCISE C, Page 14.

1. (a) -1.245; rational (b) $0.\dot{3}\dot{4}$; rational (c) $\sqrt{3}$; irrational

(d) π; irrational (e) $\sqrt{25}$; rational (f) $2\sqrt{6}$; irrational

END OF LESSON 11

LESSON RECORD FORM

0902 Math 9
Revised 11/90

FOR STUDENT USE ONLY

Date Lesson Submitted

_____

Time Spent on Lesson

_____

(If label is missing or incorrect)

File Number

_____

Lesson Number _____

FOR A.D.L.C. USE ONLY

Assigned Teacher: _____

Lesson Grading: _____

Additional Grading E/R/P Code: _____

Mark: _____

Graded by: _____

Assignment Code: _____

Date Lesson Received:

_____

Lesson Recorded _____

Student's Questions and Comments

Apply Lesson Label Here

Name

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Postal Code

Please verify that preprinted label is for correct course and lesson.

Teacher's Comments:

_____

Correspondence Teacher

St. Serv. 14-91

ALBERTA DISTANCE LEARNING CENTRE

MAILING INSTRUCTIONS FOR CORRESPONDENCE LESSONS

1. BEFORE MAILING YOUR LESSONS, PLEASE SEE THAT:

(1) All pages are numbered and in order, and no paper clips or staples are used.
(2) All exercises are completed. If not, explain why.
(3) Your work has been re-read to ensure accuracy in spelling and lesson details.
(4) The Lesson Record Form is filled out and the correct lesson label is attached.
(5) This mailing sheet is placed on the lesson.

2. POSTAGE REGULATIONS

Do **not** enclose letters with lessons.

Send all letters in a separate envelope.

3. POSTAGE RATES

First Class

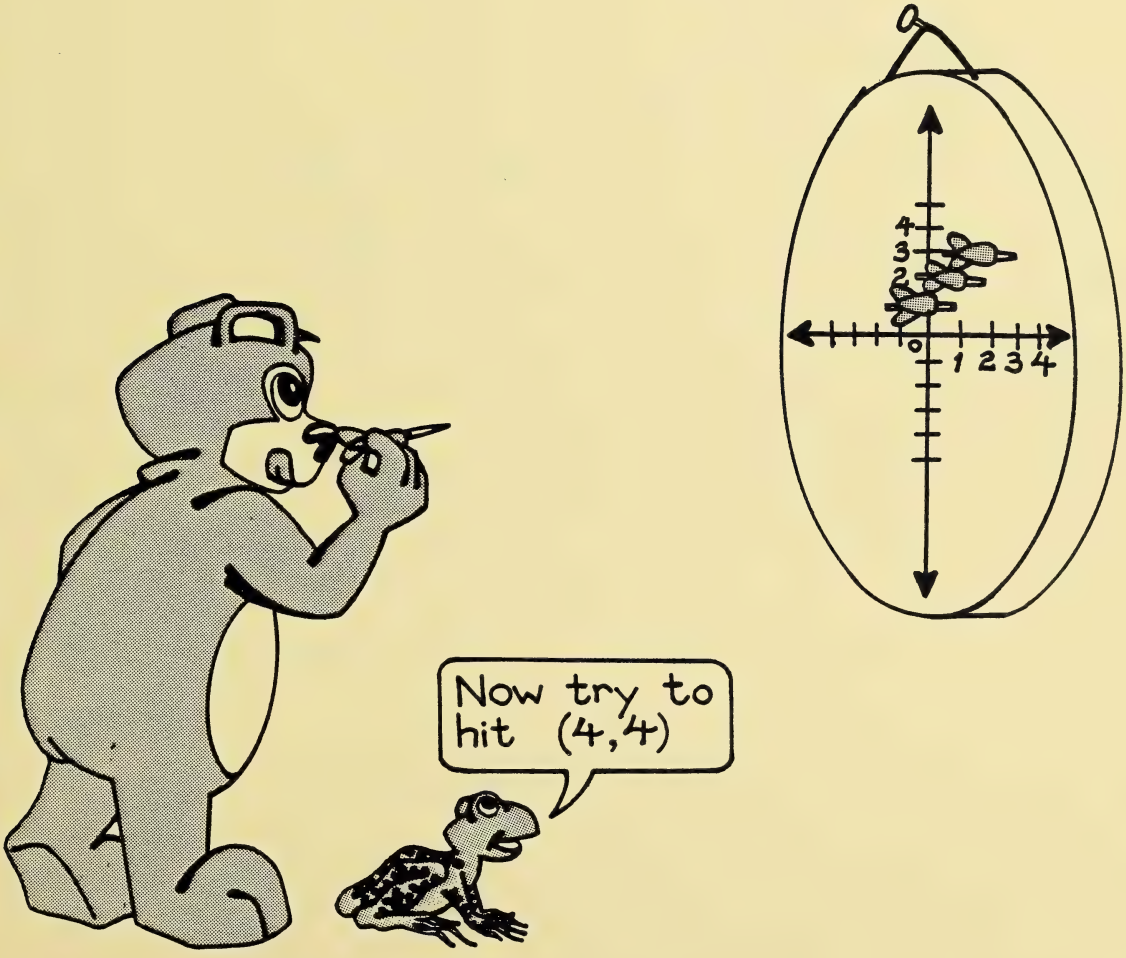
Take your lesson to the Post Office and have it weighed. Attach sufficient postage and a green first-class sticker to the front of the envelope, and seal the envelope. Correspondence lessons will travel faster if first-class postage is used.

Try to mail each lesson as soon as it has been completed.

When you register for correspondence courses, you are expected to send lessons for correction regularly. Avoid sending more than two or three lessons in one subject at the same time.

St. Serv. 9-91

Graphing Linear Equations



In this lesson you will learn about:

- identifying relations
- graphing linear equations

KEEPING SKILLFUL

EXERCISE 1

1. Use the rules of order of operations to evaluate the following expression.

   $3 \times (6 - 8) \div 2 + 4 \times 6$

2. Evaluate $2x + 9$.

   (a) if $x = -3$ (b) if $x = 8$

3. Evaluate $-3(a - 7)$.

   (a) if $a = 0$ (b) if $a = 11$

IDENTIFYING RELATIONS

Reference Read page 307 of *Holt Mathematics 3.*

Compare the following two rows of numbers. What is their relationship?

Row 1	0	1	2	3	4	5
Row 2	0	3	6	9	12	15

You should see that every number in row 2 is three times its corresponding number in row 1.

SELF-CORRECTING EXERCISE A

Identify the indicated relations by comparing row 2 to row 1.

1.

Row 1	8	9	10	11	12	13	14
Row 2	5	6	7	8	9	10	11

_____

_____

2.

Row 1	0	1	2	3	4	5	6
Row 2	-1	1	3	5	7	9	11

_____

_____

Check your answers with those on page 17 of this lesson. Correct any errors in your work.

RELATIONS

A **relation** is the connection between two sets.

Domain

The **domain** is the set from which the first members of a relation are taken.

The letter "x" is usually used to represent the members of the domain.

Range

The **range** is the set from which the second members of a relation are taken.

The letter "y" is usually used to represent the members of the range.

Example 1

The members of a relation are expressed in the following table of values.

x-values	0	1	2	3	4	5 . . .
y-values	4	5	6	7	8	9 . . .

The domain of this relation is the set of whole numbers. This can be expressed as $x \in W$.

The range is all whole numbers greater than or equal to 4. This can be expressed by $\{ y \in W, y \geq 4 \}$.

The relation shown in the table of values is $y = x + 4$, because every y-value is 4 greater than its corresponding x-value.

Example 2

x-values	. . . -2	-1	0	1	2	3 . . .
y-values	. . . -3	-1	1	3	5	7 . . .

The domain is $x \in I$.

The range is $y \in$ odd integers.

The relation is $y = 2x + 1$. (To obtain the y-value, multiply its corresponding x-value by 2 and add 1.)

SELF-CORRECTING EXERCISE B

For each of the following give the:

(a) domain,

(b) range,

(c) relation in the form of an equation.

1.

x	0	1	2	3	4	5	6	7 . . .
y	-1	0	1	2	3	4	5	6 . . .

(a) _____

(b) _____

(c) _____

2.

x	. . . -3	-2	-1	0	1	2	3 . . .
y	. . . 7	8	9	10	11	12	13 . . .

(a) _____

(b) _____

(c) _____

3.

x	0	2	4	6	8	10	12	14 . . .
y	1	7	13	19	25	31	37	43 . . .

(a) _____

(b) _____

(c) _____

Check your answers with those on page 17 of this lesson. Correct any errors in your work.

EXERCISE 2 For each of the following give the:

(a) domain,

(b) range,

(c) relation in the form of an equation.

1.

x	. . . -4	-2	0	4	6	8 . . .
y	. . . 2	1	0	-2	-3	-4 . . .

(a) _____

(b) _____

(c) _____

2.

x	0	1	2	3	4	5	6	7 . . .
y	1	6	11	16	21	26	31	36 . . .

(a) _____

(b) _____

(c) _____

3.

x	0	1	2	3	4	5	6 . . .
y	0	1	8	27	64	125	216 . . .

(a) _____

(b) _____

(c) _____

FINDING A TABLE OF VALUES

Given an equation and information on the domain and range, you can fill in a table of values.

Example 1 Given the equation $y = \frac{1}{2}x + 3$, complete the table of values. The domain is given, the range is the set of real numbers.

x	0	1	2	3	4	5	6
y							

To determine the y-values, substitute the corresponding x-values into the equation.

For $x = 0$, $y = \frac{1}{2}(0) + 3$

$y = 0 + 3$

$y = 3$

For $x = 1$, $y = \frac{1}{2}(1) + 3$

$y = \frac{1}{2} + 3$

$y = 3\frac{1}{2}$

By continuing this substitution process the table of values is completed.

x	0	1	2	3	4	5	6
y	3	$3\frac{1}{2}$	4	$4\frac{1}{2}$	5	$5\frac{1}{2}$	6

Example 2 Given the equation $y = -2x$, where $x \in R$ and $y \in R$, fill in the table of values.

Note: In this case, choose any real numbers for the x-values. Then, substitute these x-values into the equation to determine the y-values.

x	-4	-2	$-\frac{1}{2}$	0	$\frac{1}{2}$	2	4
y	8	4	1	0	-1	-4	-8

For $x = -4$: $y = -2(-4)$

$y = 8$

For $x = -2$: $y = -2(-2)$

$y = 4$

For $x = -\frac{1}{2}$: $y = -2(-\frac{1}{2})$

$y = 1$

For $x = 0$: $y = -2(0)$

$y = 0$

EXERCISE 3 Complete the following tables of values. In each case, $x \in R$ and $y \in R$. Show your work below the table.

1. $y = x^2$

x	-5	-3	-1	0	1	3
y						

2. $y = -3x + 5$

x	2	4	6	8	10
y					

3. $y = 3(x - 2)$

Remember: do the operations inside the brackets first.

x							
y							

ORDERED PAIRS

The corresponding x and y-values from a table of values can be written as ordered pairs.

(x, y) is an ordered pair.

Given the table of values:

x	1	2	3	4	5	6
y	8	9	10	11	12	13

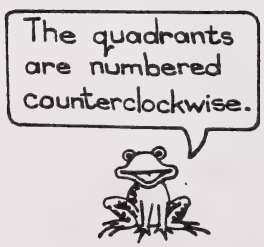
the ordered pairs are (1, 8), (2, 9), (3, 10), (4, 11), (5, 12) and (6, 13).

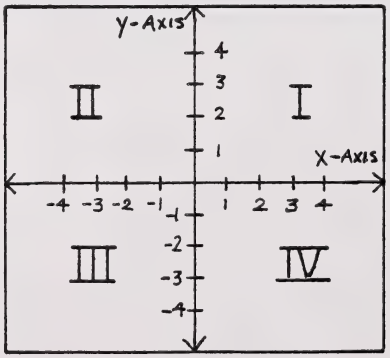
> The first member of an ordered pair is always the x-value (abscissa).
>
> The second member of an ordered pair is always the y-value (ordinate).

Ordered pairs are graphed on a Cartesian plane. This plane is formed by two perpendicular number lines called the x and y-axis.

x-axis — The x-axis is the horizontal axis. The first member of an ordered pair is graphed according to the x-axis.

y-axis — The y-axis is the vertical axis. The second member of an ordered pair is graphed according to the y-axis.





Origin

The point where the x and y-axis intersect is called the origin. The origin is represented by the point (0, 0).

Quadrants

The intersecting axes divide the Cartesian plane into four quadrants. These quadrants are numbered I, II, III, IV. The quadrants are numbered in a counter-clockwise sequence.

Note the following:

In quadrant I, the x and y-values are both positive.

In quadrant II, the x-values are negative and the y-values are positive.

In quadrant III, the x and y-values are both negative.

In quadrant IV, the x-values are positive and the y-values are negative.

SELF-CORRECTING EXERCISE C

List the ordered pairs from the table values beside the quadrant they fit into.

x	-4	-3	-2	-1	0	1	2	3	4
y	8	-6	4	-2	0	-2	4	-6	8

Quadrant I _____

Quadrant II _____

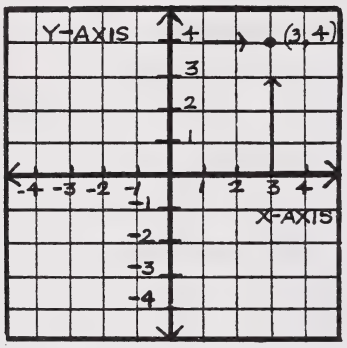
Quadrant III _____

Quadrant IV _____

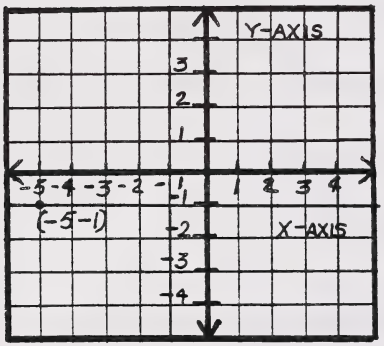
Origin _____

Check your answers with those on page 17 of this lesson. Correct any errors in your work.

GRAPHING ORDERED PAIRS

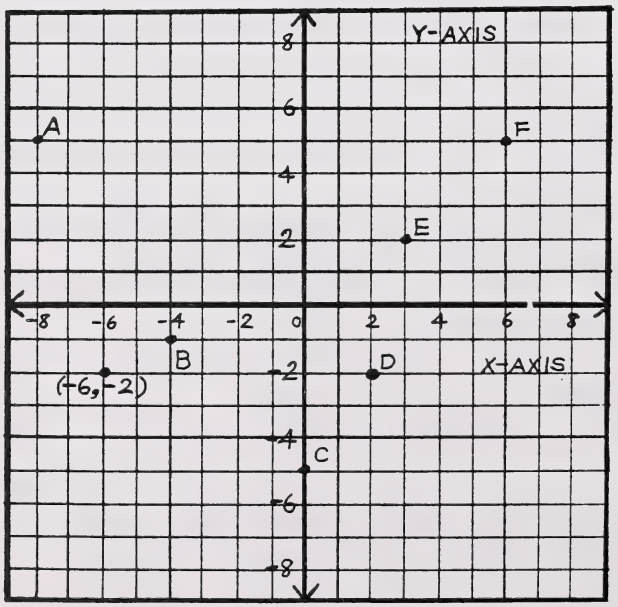


To plot the point (3, 4) on a Cartesian plane, first find +3 on the x-axis. Then, go directly up from x = +3 until you are directly across from y = +4. This is where you draw your point (3, 4).



To plot the point (-5, -1), first find x = -5 on the x-axis. Then, go directly downwards until you are across from y = -1.

EXERCISE 4 Study the graph below and answer the following questions.



1. Answer the following questions using the diagram on page 12.

   (a) Which ordered pair is associated with the point:

   A _____ D _____

   B _____ E (3, 2)

   C _____ F _____

2. Mark the point which corresponds to the following ordered pairs on the graph on page 12. Then, tell which quadrant the point is in.

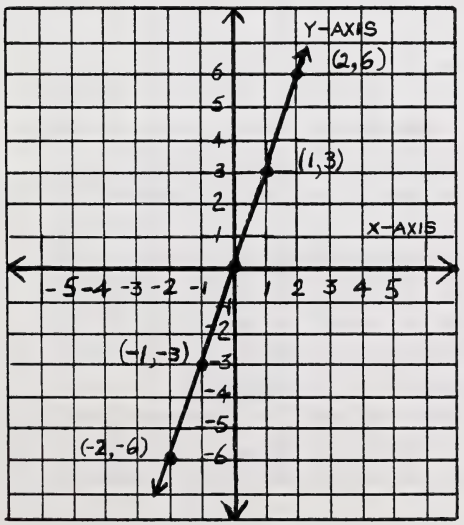
   (-6, -2) *quadrant III* (4, -1) _____

   (2, 6) _____ (-2, 7) _____

GRAPHING LINEAR EQUATIONS

To graph a linear equation such as $y = 3x$, you must first complete a table of values. Then graph the ordered pairs from this table of values onto a Cartesian plane.

Example 1 Graph $y = 3x$, where $x \in R$.



x	-2	-1	0	1	2
3x	-6	-3	0	3	6

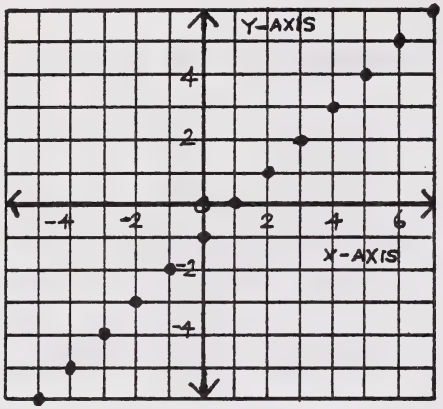
For $x = -2$: $y = 3(-2) = -6$

For $x = -1$: $y = 3(-1) = -3$

And so on, for other values of x.

Notice that after the points were plotted, a line was drawn through the points. Recall from Lesson 12, that real numbers are graphed by solid lines.

Example 2 Graph the linear equation $y = x - 1$, where $x \in I$.



x	-3	-2	-1	0	1	2
x - 1	-4	-3	-2	-1	0	1

Notice that the points are in a straight line, *but in this case you do not draw in that line.*

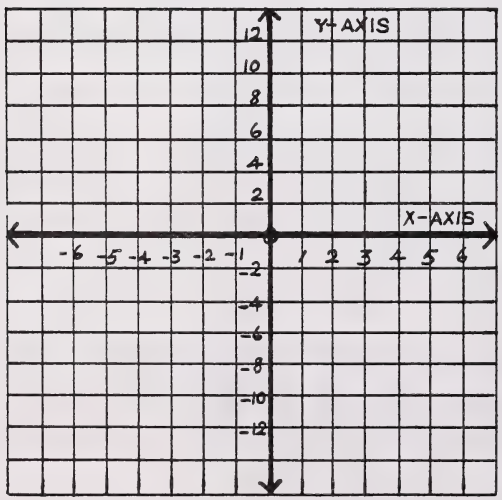
Recall from Lesson 11, that integers are not dense (nor are whole numbers). Thus, the graph consists of individual points. The points are continued to the ends of the graph to show that they continue to infinity.

$y = 3x$ and $y = x - 1$ are both called linear equations because when graphed, the points will lie on a straight line.

SELF-CORRECTING EXERCISE D

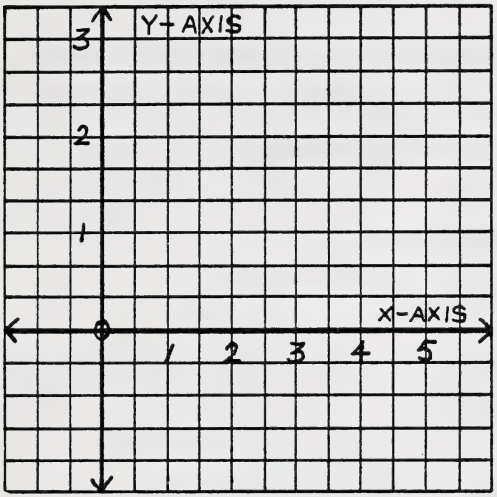
Fill in a table of values and graph the following linear equations.

1. $y = 4x - 3$, $x \in R$



x						
4x - 3						

2. $y = \frac{1}{3}x,\ x \in W$

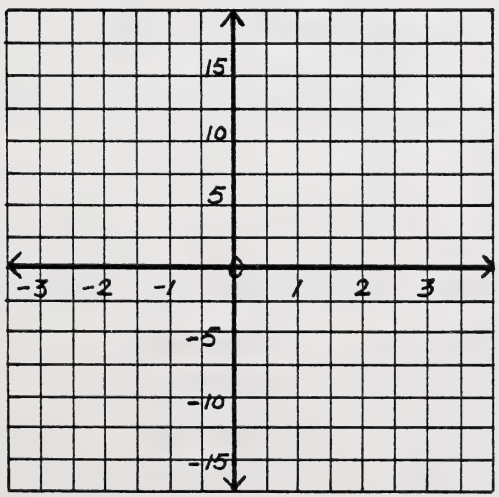


x							
$\frac{1}{3}x$							

Check your answers with those on pages 18 and 19 of this lesson. Correct any errors in your work.

EXERCISE 5 For each of the following linear equations, fill in a table of values and draw a graph.

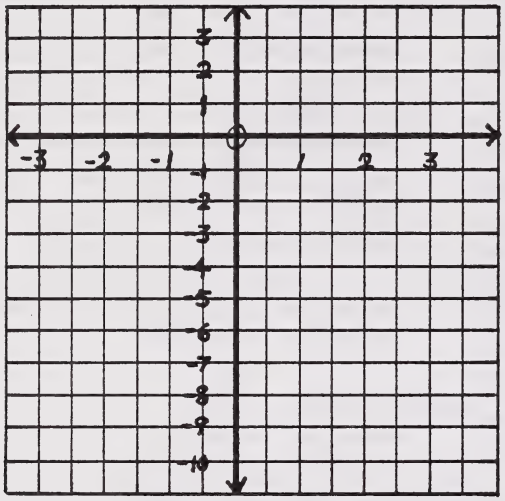
1. $y = -5x,\ x \in R$



x							
-5x							

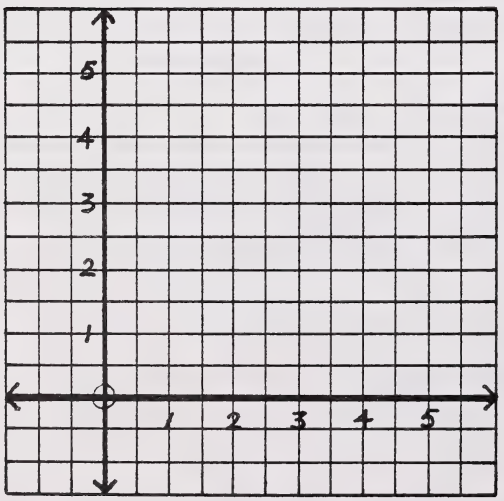
EXERCISE 5 (continued)

2. $y = x - 6$, $x \in I$



x						
x - 6						

3. $y = \frac{1}{2}x + 3$, $x \in W$



x							
$\frac{1}{2}x + 3$							

KEY TO SELF-CORRECTING EXERCISES IN LESSON 14

SELF-CORRECTING EXERCISE A, page 3.

1. Every number in row 2 is three less than its corresponding number in row 1.

2. To obtain the numbers in row 2, multiply the corresponding number in row 1 by two and subtract one.

SELF-CORRECTING EXERCISE B, page 5.

1. (a) $x \in W$

   (b) $\{y \in I, y \geq -1\}$

   (c) $y = x - 1$ *Every y-value is one less than the corresponding x-value.*

2. (a) $x \in I$

   (b) $y \in I$

   (c) $y = x + 10$ *Every y-value is 10 greater than the corresponding x-value.*

3. (a) $x \in$ even whole numbers

   (b) $y \in \{1, 7, 13, 19, 25, 31, 37, 43 \ldots\}$

   Note: Since the range is not a general set, the members of the set are listed.

   (c) $y = 3x + 1$ *To obtain the y-value, multiply the x-value by 3 and add 1.*

SELF-CORRECTING EXERCISE C, page 11.

Quadrant I	(2, 4)(4, 8)
Quadrant II	(-4, 8)(-2, 4)
Quadrant III	(-3, -6)(-1, -2)
Quadrant IV	(1, -2) (3, -6)
Origin	(0, 0)

SELF-CORRECTING EXERCISE D, pages 14 and 15.

1.

x	-2	-1	0	1	2	3
4x - 3	-11	-7	-3	1	5	9

If $x = -2$:

$y = 4(-2) - 3$

$y = -8 - 3$

$y = -11$

If $x = -1$:

$y = 4(-1) - 3$

$y = -4 - 3$

$y = -7$

If $x = 0$:

$y = 4(0) - 3$

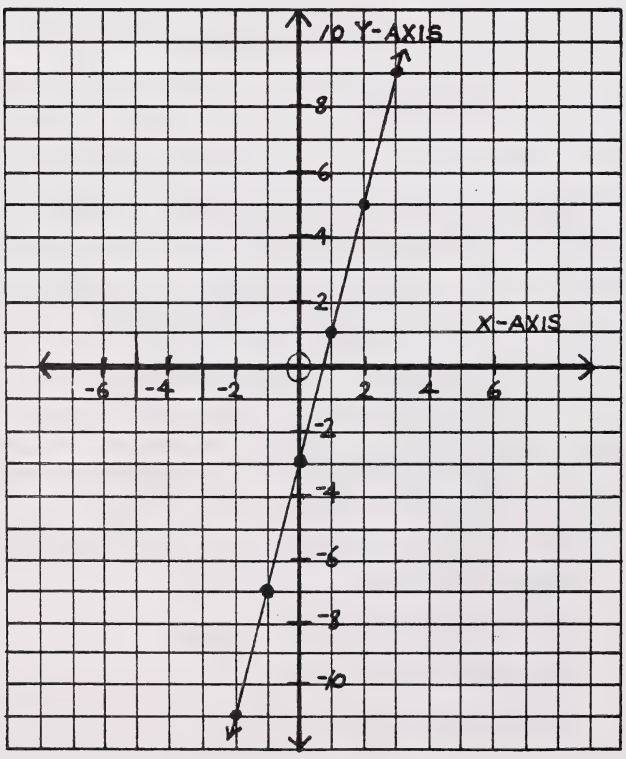
$y = -3$

If $x = 1$:

$y = 4(1) - 3$

$y = 1$

And so on, . . .



Note: A solid line is drawn because $x \in R$.

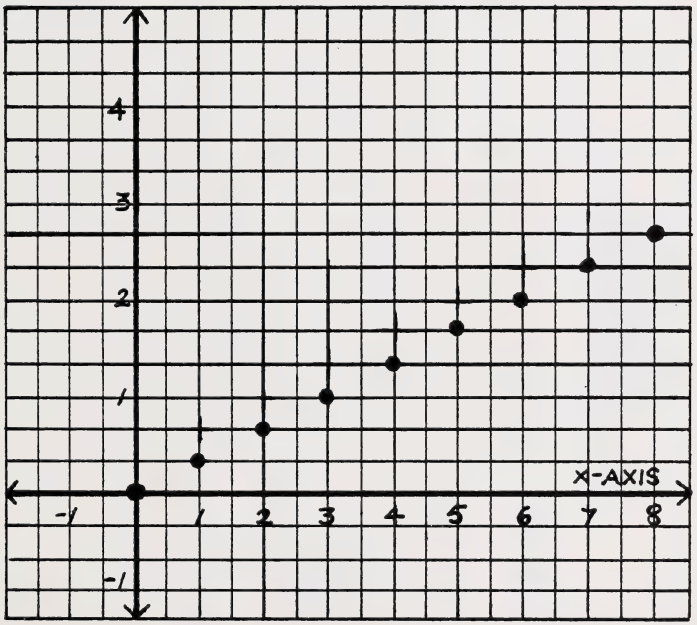
2.

x	0	1	2	3	4	5	6
$\frac{1}{3}x$	0	$\frac{1}{3}$	$\frac{2}{3}$	1	$1\frac{1}{3}$	$1\frac{2}{3}$	2

If $x = 0$: $y = \frac{1}{3}(0) = 0$

If $x = 1$: $y = \frac{1}{3}(1) = \frac{1}{3}$

If $x = 2$: $y = \frac{1}{2}(2) = \frac{2}{3}$ And so on, . . .



Since $x \in W$ you can only use the whole numbers for the x-values.

Also, whole numbers are not dense. Thus, the graph consists of individual points that are not connected by a line. Notice that all possible points are drawn in, up to the end of the graph paper.

END OF LESSON 12

LESSON RECORD FORM

0902 Math 9
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(1) All pages are numbered and in order, and no paper clips or staples are used.
(2) All exercises are completed. If not, explain why.
(3) Your work has been re-read to ensure accuracy in spelling and lesson details.
(4) The Lesson Record Form is filled out and the correct lesson label is attached.
(5) This mailing sheet is placed on the lesson.

2. POSTAGE REGULATIONS

Do **not** enclose letters with lessons.

Send all letters in a separate envelope.

3. POSTAGE RATES

First Class

Take your lesson to the Post Office and have it weighed. Attach sufficient postage and a green first-class sticker to the front of the envelope, and seal the envelope. Correspondence lessons will travel faster if first-class postage is used.

Try to mail each lesson as soon as it has been completed.

When you register for correspondence courses, you are expected to send lessons for correction regularly. Avoid sending more than two or three lessons in one subject at the same time.

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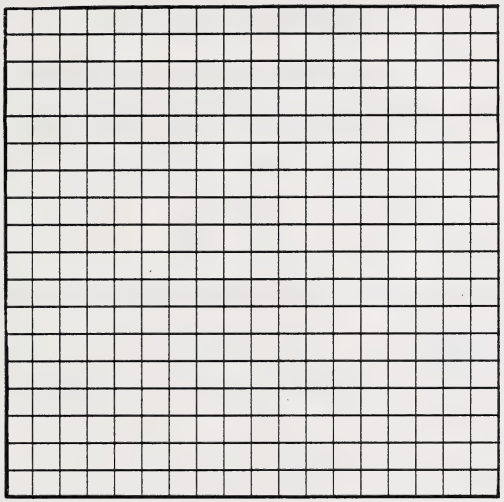
FORMULAS

In this lesson you will learn about:

- evaluating algebraic expressions
- evaluating formulas
- rearranging formulas to find a particular variable

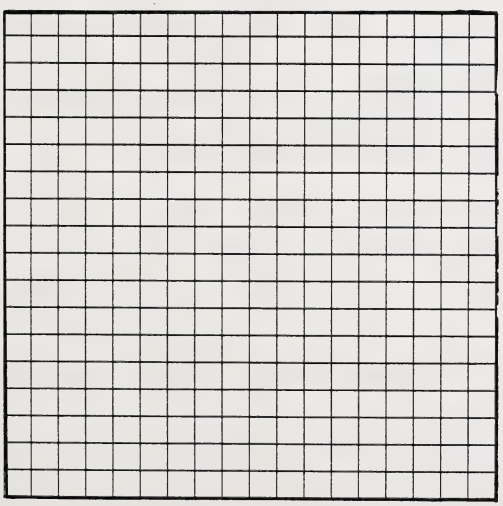
EXERCISE 1 Fill in a table of values and graph the following linear equations.

1. $y = 3x - 2,\ x \in R$



x					
$3x - 2$					

2. $y = \frac{1}{4}x,\ x \in W$



x					
$\frac{1}{4}x$					

EVALUATING EQUATIONS

In Lesson 12 you studied the graphing of linear equations. In order to graph the equations, you set up a table of values. For example:

$y = 3x - 4$

x	-2	-1	0	1	2
y	-10	-7	-4	-1	2

To get each value of y, you substituted a number for x in the equation.

x and y are called **variables** because they can take on many different values.

EVALUATING ALGEBRAIC EXPRESSIONS

Variables appear in many different kinds of algebraic expressions. In every case, numbers may be substituted for the variables and a value for the expression obtained.

This process is called **evaluating the expression.**

In order to evaluate expressions properly, you must use the Order of Operations rules given in Lesson One and on pages 8 and 9 of *Holt Mathematics 3*.

If you have forgotten these rules, please review them before going on with this lesson.

Example 1 Evaluate $ab + bc + 3ac$, for $a = 2$, $b = 3$, and $c = 0$.

$ab + bc + 3ac$

$= (2)(3) + (3)(0) + 3(2)(0)$ ← *Substitute.*

$= 6 + 0 + 0$ ← *Do multiplication first.*

$= 6$ ← *Add to get the answer.*

Example 2 Evaluate $\frac{x + y}{y - 2x} - \frac{2xy}{x + 2y}$, for $x = -1$ and $y = 3$.

$$\frac{x + y}{y - 2x} - \frac{2xy}{x + 2y}$$

Treat the expression as though it were written as follows:

$$\left(\frac{x + y}{y - 2x}\right) - \left(\frac{2xy}{x + 2y}\right)$$

$$= \left(\frac{(-1) + 3}{3 - 2(-1)}\right) - \left(\frac{2(-1)(3)}{(-1) + 2(3)}\right) \leftarrow \textit{Substitute.}$$

$$= \left(\frac{(-1) + 3}{3 + 2}\right) - \left(\frac{-6}{(-1) + 6}\right) \leftarrow \textit{Do the multiplication inside the brackets.}$$

Notice that each numerator and denominator is evaluated separately.

$$= \left(\frac{2}{5}\right) - \left(\frac{-6}{5}\right) \longleftarrow \textit{Do the addition inside the brackets.}$$

$$= \frac{2}{5} + \frac{6}{5} \longleftarrow \textit{Remove the brackets and add.}$$

$$= \frac{8}{5}$$

Example 3 Evaluate $2x^2 - 3y^2 + 5z^2$, for $x = 3$, $y = -2$ and $z = 1$.

$$2x^2 - 3y^2 + 5z^2$$

$$= 2(3)^2 - 3(-2)^2 + 5(1)^2 \qquad \textit{Substitute.}$$

$$= 2(3)(3) - 3(-2)(-2) + 5(1)(1) \qquad \textit{Expand and multiply.}$$

$$= 18 - 12 + 5 \qquad \textit{Add and subtract.}$$

$$= 11$$

SELF-CORRECTING EXERCISE A

If $a = 1$, $b = -2$, $c = 3$, evaluate:

1. $a + 4b - 3c$

2. $\frac{3a + 12b}{4c}$

3. $a^2 + b^2 - 2c^2$

Check your answers with those on page 19 of this lesson.
Correct any errors in your work.

EXERCISE 2 1. Evaluate for $a = -1$, $b = 2$, $c = 3$.

(a) $6b + (c - 3a)$

(b) $\frac{ac}{b} + \frac{bc}{3a} - \frac{abc}{2}$

2. Evaluate for $p = -5$, $q = 2$, $r = -1$.

(a) $(p + q + r)^2$

(b) $\frac{pr - q}{p - 2r}$

FORMULAS

Through constant usage, certain letters have gradually begun to represent the same unknown variables. You are already familiar with the use of:

A to represent area
ℓ to represent length
P to represent perimeter

The equations:

$P = 2\ell + 2w$, and

$A = \ell w$

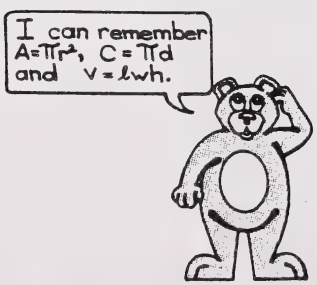
express general relationships connecting the variables P, ℓ, w and A.

Such equations, which have been developed for particular uses, are called **formulas**.

Care must be taken, however, to see that the units used are correct. For instance, you cannot find area by multiplying 6 cm by 2 m. Similarly, you cannot find perimeter by adding millimetres and kilometres.

You should be able to recall several other formulas which you have already used.

Example 1: The distance travelled (d), at a velocity (v) for a certain time is $d = vt$. Find the distance travelled at 100 km/h for 5 hours.

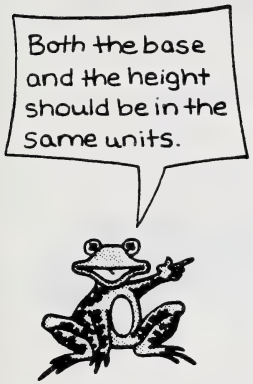


$d = ?$ $v = 100$ $t = 5$

$d = vt$
$= 100 \times 5$
$= 500$

The distance is 500 km.

Example 2 The area of a triangle is given by the formula $A = \frac{1}{2}bh$, where b is the base and h is the height. Find the area of a triangle whose base is 0.18 m and whose height is 33 cm.



A = ? b = 0.18 m, h = 33 cm
A = ? b = 18 cm, h = 33 cm

$$A = \frac{1}{2}bh$$

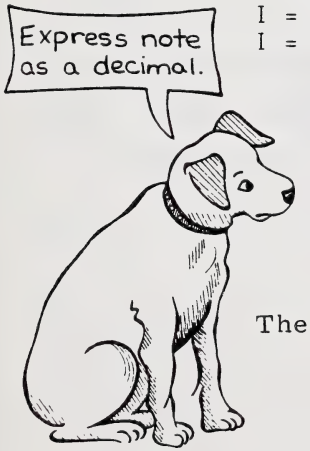
$$= \frac{1}{\cancel{2}} \times \cancel{18}^{9} \times 33$$

$$= 9 \times 33$$

$$= 297$$

The area is 297 cm^2.

Example 3 The simple interest earned on a sum of money (P), at an annual rate (r) for t years, is I = Prt. Find the interest earned on $800 at 15% for 3 years.



I = ? P = 800, r = 15%, t = 3
I = ? P = 800, r = 0.15, t = 3

$$I = Prt$$

$$= 800 \times 0.15 \times 3$$

$$= 120 \times 3$$

$$= 360$$

The interest is $360.

NOTE: Every formula question should end with a sentence answer to convey this meaning.

EXERCISE 3 Evaluate as directed in the following formulas.

(a) Copy the formula, make the substitutions, and find the value of the unknown variable.

(b) Write a sentence which answers the question, as shown in the examples on page 8.

1. The perimeter of a rectangle is found by the formula $P = 2l + 2w$, where l and w represent the length and width. Find the perimeter of a rectangle whose length is 14.6 m and whose width is 3.8 m.

2. The circumference (C) of a circle is given by $C = 2\pi r$, where $\pi = 3.14$ (approximately) and r is the radius. Find the circumference, in kilometres, of a circle whose radius is 3.5 km.

EXERCISE 3 (continued)

3. $F = \frac{13\ mv^2}{r}$ is the formula for the force (F), in newtons, exerted when an object of mass (m) kilograms moves in a circular path of radius (r) metres at a speed of (v) kilometres per hour.

Find the force which seems to be pulling a 1000 kg car off an icy curve of radius 50 metres, when the car is going 60 km/h.

4. $v = u + at$ gives the final speed (v) of a moving object where the initial speed is u, the acceleration is a, and the time travelled is t.

Find the final speed of a car after 0.4 hours of acceleration at 200 km/h^2, if its initial speed was 20 km/h.

REARRANGING FORMULAS

You have already spent a considerable amount of time studying the solution of equations. Remember that a formula is an equation that usually has more letters than numbers.

Therefore, to rearrange formulas you use the same methods you used for equations.

Basic Rules for Equations

1. The two sides of an equation must always remain equal.
2. Whatever you do to one side of an equation you must do to the other side as well.

Rule 2 is necessary to preserve Rule 1. That is, Rule 1 is the reason for always using Rule 2.

In science and the trades it is quite common to rearrange a formula for a specific use.

Example 1 The interest formula is:

$$\text{Interest} = \text{Principal} \times \text{Rate} \times \text{Time}$$

$$\text{or,} \quad I = Prt$$

To find I, you put in numbers for P, r, and t, and multiply.

It is frequently useful to rearrange such a formula. Perhaps an employee of an investment firm has to find the principal required to produce a certain amount of interest over various periods of time at different rates.

$I = Prt$ is the same as $Prt = I$.

$$Prt = I$$

$$\frac{\cancel{Prt}^{1}}{\cancel{rt}_{1}} = \frac{I}{rt} \qquad \textit{Divide both sides by } rt \textit{ to isolate } P.$$

$$P = \frac{I}{rt} \qquad \textit{Simplify.}$$

You now have a formula for principal.

Example 2 $A = P + I$ is a formula that gives the amount of money (A) that an investor accumlates on a principal (P) when the interest is I.

Rearrange the formula to find I.

$$P + I = A$$

$$P + I - P = A - P \quad \text{Subtract P from both sides to isolate I.}$$

$$I = A - P \quad \text{Simplify.}$$

Example 3 Sometimes, there is a variable in the denominator of of a formula. Usually it is easier to rearrange the formula if you eliminate the denominator.

$I = \frac{V}{R}$ is a formula in electricity.

(a) Rearrange the formula to find V.

$$\frac{V}{R} = I$$

$$\frac{VR}{R} = IR \quad \text{Multiply both sides by R to eliminate the denominator.}$$

$$V = IR \quad \text{Simplify.}$$

(b) Rearrange the formula to find R.

$$I = \frac{V}{R}$$

$$IR = \frac{VR}{R} \quad \text{Multiply both sides by R to clear the denominator}$$

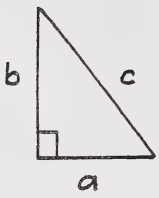
$$IR = V \quad \text{Simplify.}$$

$$\frac{IR}{I} = \frac{V}{I} \quad \text{Divide both sides by I to isolate R.}$$

$$R = \frac{V}{I} \quad \text{Simplify.}$$

Example 4 Sometimes the formula has variables that are squared.

$c^2 = a^2 + b^2$ is the formula for the Pythagorean Theorem. c, ab and b are sides of a right-angled triangle. Rearrange the formula to find a.



$$a^2 + b^2 = c^2$$

$$a^2 + b^2 - b^2 = c^2 - b^2$$ *First find a^2.*

$$a^2 = c^2 - b^2$$ *Take the square root of both sides.*

$$\sqrt{a^2} = \sqrt{c^2 - b^2}$$

$$a = \sqrt{c^2 - b^2}$$

SELF-CORRECTING EXERCISE B

1. $V = Bh$ gives the volume (V) of a box, when B is the area of its base and h is its height.

   Rearrange the formula to find B.

2. $S = C + G$ is a business formula showing that the Selling Price (S) is the sum of the Cost (C) and the Gain (G).

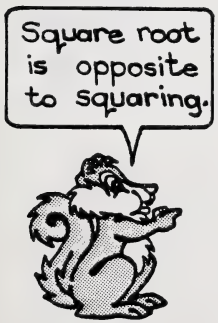
   Rearrange the formula to find G.

SELF-CORRECTING EXERCISE B (continued)

3. $d = \frac{m}{v}$ is the formula to find the density (d) of a substance whose mass (m) and volume (v) are known.

   (a) Rearrange the formula to find m.

   (b) Rearrange the formula to find v.



4. $A = \pi r^2$ is the formula for the area of a circle with radius r.

   Rearrange the formula to find r.

5. $L = C - S$ is a business formula showing that the loss (L) is the difference between the cost (C) and the selling price (S).

   (a) Rearrange the formula to find C.

   (b) Rearrange the formula you used in part (a) to find S.

Check your answers with those on pages 19 and 20 of this lesson. Correct any errors in your work.

EXERCISE 4

1. $D = VT$ is the formula for the distance (D) travelled at an average velocity (V) for a time (T).

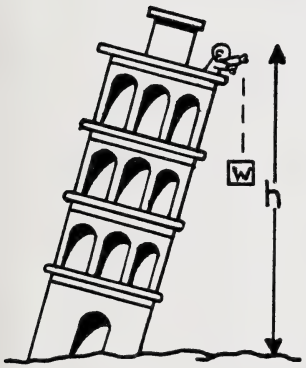
   (a) Rearrange the formula to find T.

   (b) Find T when $D = 150$ km and $V = 65$ km/h. (Round to one decimal place.)

EXERCISE 4 (continued)

2. $P = \frac{Wh}{t}$ is a formula used in mechanics to find the power (P) generated when a weight (W) falls a distance (h) in a time (t).

(a) Rearrange the formula to find t.



(b) Rearrange the formula to find h.

(c) Find t, in seconds, when P = 343 kW (kilowatts), W = 100N (newtons), and h = 35 m.

3. $C = A - L$ is a bookkeeping formula stating that a firm's capital (C) equals its assets (A) minus its liabilities (L).

   (a) Rearrange the formula to find A.

   (b) Rearrange the formula you found in part (a) to find L.

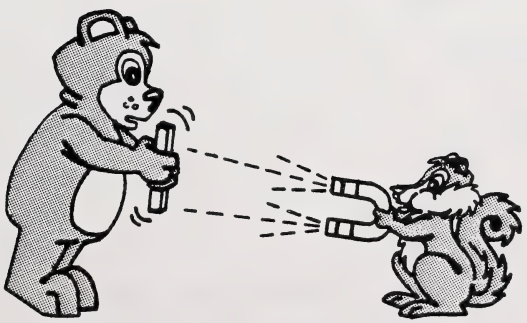
   (c) Find L when $C = \$270\ 000$ and $A = \$186\ 000$.

EXERCISE 4 (continued)

5. $A = \frac{k}{d^2}$ is a formula for the attraction (A) between a magnet and an iron bar, when the distance between them is d mm. k is a constant number (similar to π).

(a) Rearrange the formula to find d.

(b) Find d if k = 648 and A = 72 dyn.



KEY TO SELF-CORRECTING EXERCISES IN LESSON 15

SELF-CORRECTING EXERCISE A, page 5.

1. $a + 4b - 3c$

$= 1 + 4(-2) - 3(3)$

$= 1 - 8 - 9$

$= -7 - 9$

$= -16$

2. $\frac{3a + 12b}{4c}$

$= \frac{3(1) + 12(-2)}{4(3)}$

$= \frac{3 - 24}{12}$

$= \frac{-21}{12}$

$= \frac{-7}{4}$

3. $a^2 + b^2 - 2c^2$

$= (1)^2 + (-2)^2 - 2(3)^2$

$= (1)(1) + (-2)(-2) - 2(3)(3)$

$= 1 + 4 - 18$

$= 5 - 18$

$= -13$

SELF-CORRECTING EXERCISE B, pages 13, 14 and 15.

1. $V = Bh$

$Bh = V$

$\frac{Bh}{h} = \frac{V}{h}$

$B = \frac{V}{h}$

2. $S = C + G$

$C + G = S$

$C + G - C = S - C$

$G = S - C$

SELF-CORRECTING EXERCISE B, pages 13, 14 and 15 (continued).

3. (a) $d = \frac{m}{v}$

$vd = \frac{m\cancel{v}^{1}}{\cancel{v}_{1}}$

$vd = m$

$m = vd$

(b) $d = \frac{m}{v}$

$vd = \frac{m\cancel{v}^{1}}{\cancel{v}_{1}}$

$vd = m$

$\frac{v\cancel{d}^{1}}{\cancel{d}_{1}} = \frac{m}{d}$

$v = \frac{m}{d}$

4. $A = \pi r^2$

$\frac{A}{\pi} = \frac{\cancel{\pi}^{1} r^2}{\cancel{\pi}_{1}}$

$\frac{A}{\pi} = r^2$

$\sqrt{\frac{A}{\pi}} = \sqrt{r^2}$

$\sqrt{\frac{A}{\pi}} = r$

5. (a) $L = C - S$

$C - S = L$

$C - S + S = L + S$

$C = L + S$

(b) $L = C - S$

$L + S = C$

$S = C - L$

END OF LESSON 13

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(3) Your work has been re-read to ensure accuracy in spelling and lesson details.
(4) The Lesson Record Form is filled out and the correct lesson label is attached.
(5) This mailing sheet is placed on the lesson.

2. POSTAGE REGULATIONS

Do not enclose letters with lessons.

Send all letters in a separate envelope.

3. POSTAGE RATES

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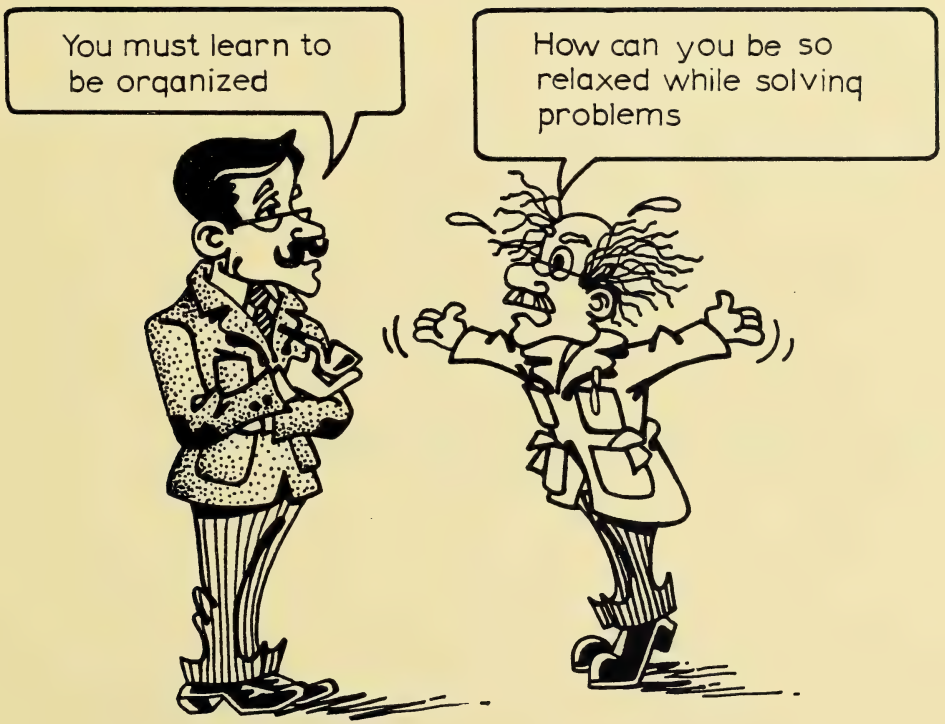
St. Serv. 9-91

Problem Solving II



In this lesson you will learn about:

- problem solving
- money problems
- age problems

Keeping Skillful

Exercise 1: Solve the following problems. Show all the steps.

(a) Two numbers differ by 14. $\frac{1}{5}$ of the larger number plus $\frac{1}{3}$ of the smaller number is 14. Find the numbers.

(b) The length of the sides of an isosceles triangle is 3 less than twice the base. If the perimeter is 54 m, find the length of the base.

In Lesson 9 you learned a method of problem solving using algebra. Two types of problems were discussed - number problems and measurement problems. In this lesson money problems and age problems will be discussed.

Money Problems

In doing money problems you must remember that the total cost of purchasing a number of items is found by multiplying the cost of one item by the number of items bought for example:

5 books @ 1.95 = 5 x 1.95 = 9.75

This symbol means "at"

Self-Correcting Exercise A

Find the total cost of the following:

(a) 5 pencils @ $.20 _____

(b) 3 records @ $9.95 _____

(c) 6 cans of pop @ $.75 _____

(d) 3 sandwiches @ $1.99 _____

(e) 3 skirts @ $45 and 5 blouses @ $32. _____

(f) 2 comics @ .60 and 4 magazines @ $1.95 _____

(g) 5 pens @ $.80 and 6 notebooks @ $1.95 _____

Check your answers with those on page 20 of this lesson. Correct any errors in your work.

Money Problems Using Algebra

Some money problems are more easily solved using algebra.

Example:

A pair of track shoes costs twice as much as a pair of running shoes. If 4 pairs of track shoes and 8 pairs of running shoes cost $520.00, find the cost of each type.

As in Lesson 9, you should read the problem a number of times. First find the main idea, then the implied details, and finally find the relevant details. A chart may help you to organize the details.

Solution:

Let the cost of the running shoes be x
Let the cost of the track shoes be 2x

Items	Number	@	Total
running shoes	8	x	8x
track shoes	4	2x	8x
			$520

$8x + 8x = 520$

$16x = 520$

$\frac{16x}{16} = \frac{520}{16}$

$x = 32.50$

$2x = 65.00$

Running shoes cost $32.50
Track shoes cost $65.00

Check:

$8 \times 32.50 = 260$

$4 \times 65.00 = 260$

$260 + 260 = 520$

Self-Correcting Exercise B

Fill in the following tables using the facts given in the problems.

1. Jill bought some records on sale. She bought 11 records altogether. The records cost a total of $76. If some records were $8 and the rest $6, how many of each type of record did she buy?

Items	Number	@	Total ($)
$8 records			
$6 records			

2. Shelley can buy 1 can of pop and 2 hotdogs for $3. If a hotdog costs twice the cost of a can of pop, find the cost of a hotdog and the cost of a can of pop.

Items	Number	@	Total ($)
hotdog			
can of pop			

3. Kevin spent $408 to purchase 9 bats for the baseball team. Some of the bats cost $38 each while the others cost $49 each. How many of each type of bat did he buy?

Items	Number	@	Total ($)
$38 bats			
$49 bats			

Check your answers with those on page 20 of this lesson. Correct any errors in your work.

Exercise 2: Solve the following problems. Show all steps.

1. A golf ball costs $\frac{1}{5}$ of the price of a baseball. If 40 golf balls and 15 baseballs cost \$172.50, find the price of each kind of ball.

2. Adult tickets cost \$0.50 more than student tickets. If 150 adult tickets and 100 student tickets were sold for a total of \$275, how much do adult tickets cost?

3. Yesterday Sally and George paid $6.00 for four cans of pop and two burgers. If a burger costs twice as much as a can of pop, what was the cost of a can of pop?

4. Rhonda bought 60 cans of soup for $12.06. If some cans cost $.18 and the rest $.25 each, how many cans did she buy at each price?

Coin Problems

Coin puzzles are very popular. These can be solved by using the same skills needed for money problems.

Self-Correcting Exercise C

Find the value in cents for the following:

(a) 5 dimes _____

(b) x dimes _____

(c) k nickels _____

(d) (k - 3) dimes _____

(e) b dimes, (3b + 1) nickels _____

(f) x quarters, (x + 2) nickels _____

(g) d dimes, (d + 1) quarters _____

Check your answers with those on page 21 of this lesson. correct any errors.

Finding Equations for Coin Problems

Example 1:

If 25 coins were used to pay a bill of $1.40 and only dimes and nickels were used, how many of each coin were used?

Items	Number	@	Total
dimes	x	10	10x
nickels	25 - x	5	5 (25 - x)

$1.40 = 140 cents

$10x + 5(25 - x) = 140$

$10x + 125 - 5x = 140$

$5x + 125 = 140$

$5x + 125 + (-125) = 140 + (-125)$

$5x = 15$

$x = 3$

$25 - x = 25 - 3 = 22$

3 dimes and 22 nickels were used.

Check:

$3 \times 10 = 30$

$22 \times 5 = 110$

$30 + 110 = 140$

Self-Correcting Exercise D

Fill out a chart and then write equations for each of the following problems. Use x for the variable and express units in cents.

(a) I have twice as many dimes as I have quarters. If I have $9, how many quarters do I have?

Items	Number	@	Total

Equation: _____

(b) The number of dimes I have is 10 more than twice the number of nickels. If the value of my coins is $9, how many dimes do I have?

Items	Number	@	Total

Equation: _____

(c) I have fifteen more dimes than quarters. I have $8.85 in dimes and quarters. How many of each type of coin do I have?

Items	Number	@	Total

Equation: _____

(d) I have 36 more cents than twice the number of nickels. If the value of my coins is $1.62, how many nickels do I have?

Items	Number	@	Total

Equation: _____

Check your answers with those on page 21 of this course. Correct your errors.

Exercise 3

Solve the following problems. Show all steps.

(a) I have 20 more nickels than I have dimes. If the sum of money I have is $8.50, how many coins of each type do I have?

(b) I have four times as many dimes as quarters. If the total value of the coins was $20.80, how many dimes and how many quarters do I have?

(c) I have \$6.25 in nickels and quarters. The number of quarters is three more than twice the number of nickels. How many quarters do I have?

(d) I have \$3.00 in dimes and nickels. The number of nickels in 8 less than twice the number of dimes. How many dimes do I have?

Age Problems

Age problems are sometimes confusing because they are written like riddles. It helps to organize the facts in the problem. First decide which people are being compared in the problem. Then decide at what times their ages are being compared.

Example

I am now nine times the age of my son. In 3 years I will be five times as old as my son. Find our ages at the present time.

Solution:

1. Notice that the people being compared are a parent and a son. the times at which their ages are being compared are now and in three years time.

2. It helps to organize these facts in a table.

	Now	In 3 years
Parent's age		
son's age		

3. Once you have the table look for a relationship between the people's ages.

   "I am now nine times the age of my son".

   Let the son's age be x

   Let the parent's age be 9x

4. Fill in the table

	Now	In 3 Years
Parent's age	9x	
son's age	x	

5. The second column can be filled next

If the parent's age is now 9x years, in three years the parent will be 9x + 3.
If the son's age is now x years, in three years the son will be x + 3.

	Now	In 3 years
Parent's age	9x	9x + 3
Son's age	x	x + 3

6. Next look for the remaining fact.

"In 3 years I will be 5 times as old as my son."

Use this fact to write an equation. (Since the relation says in three years, use the variables under this heading.)

$9x + 3 = 5(x + 3)$

7. Solve the equation.

$9x + 3 = 5(x + 3)$
$9x + 3 = 5x + 15$
$9x + 3 + (-3) = 5x + 15 + (-3)$
$9x = 5x + 12$
$9x + -5x = 5x + -5x + 12$
$4x = 12$
$\frac{4x}{4} = \frac{12}{4}$
$x = 3$
$9x = 27$

8. Write a statement

The parent is 27 and the son is 3

9. Check

$3 \times 9 = 27$
$27 + 3 = 30$
$3 + 3 = 6$
$6 \times 5 = 30$

Self-Correcting Exercise E

1. Today I am n years old.

   (a) What will be my age one year from now? _____

   (b) What was my age 8 years ago? _____

   (c) What will be my age in 9 years? _____

2. Today Sam is (2n + 1) years old.

   (a) What will be Sam's age in 3 years? _____

   (b) What was Sam's age five years ago? _____

   (c) What will be Sam's age next year? _____

3. Joan is n years old. Find Barb's age if Barb is:

   (a) twice Joan's age _____

   (b) 6 years older than Joan _____

   (c) presently 2 years more than 4 times Joan's age. _____

Check your answers with those on page 22 of this course. Correct your errors.

Exercise 4

1. Fill in the tables for each of the following problems.

   (a) Shirley is presently 2 years more than 4 times Rose's age.

	Now	In 12 years
Shirley		
Rose		

(b) The sum of Harry's and Jean's present ages is 30 years.

	Now	5 years ago
Harry		
Jean		

(c) My age increased by my son's age is 48 years.

	Now	Last year
Parent		
my son		

Self-Correcting Exercise F

Write equations using these tables and facts.

(a)

	Now	In 15 years
Parent's age	x	x + 15
son's age	48 - x	63 - x

In 15 years my son's age increased by 4 times my age will be 234 years.

Equation: _____

(b)

	Now	In 5 years
son's age	x	x + 5
parent's age	50 - x	55 - x

In 5 years my father will be twice my age.

Equation: _____

(c)

	Now	3 years ago
Susan	3n	3n - 3
Paul	n	n - 3

3 years ago Susan was 4 times as old as Paul.

Equation: _____

(d)

	Now	In 30 years
daughter's age	x	x + 30
parent's age	50 - x	80 - x

In 30 years, 3 times the parent's age increased by the daughter's age will be 242 years.

Equation: _____

Check your answers with those on page 22 of this course. Correct your errors.

Exercise 5

Solve the following problems. Show all steps.

1. John is presently 12 years older than Bill. In 10 years, twice John's age will be equal to 3 times bill's age. How old are they today.

2. Brad is presently 9 years older than Mary. In 8 years, 6 times Brad's age will equal 7 times Mary's age. How old are they at the present time?

3. Jason's father is 27 years older than Jason. Five years ago Jason's father was 3 years more than 5 times Jason's age. What are their present ages?

4. Ten years from now, Jane will be just 7 years short of being twice as old as she is now. Find Jane's present age.

EXERCISE 6 For each of the following put the letter of the equation that best fits in the blank provided.

_____ 1. Twice Mary's age 4 years ago equals her present age plus 8 years. How old is Mary?

a. $2(n - 4) = n + 8$
b. $2n - 4 = n + 8$
c. $2(n + 8) = n - 4$
d. $2n - 8 = 8 + 4$

_____ 2. Bill has \$3.10 in nickels and dimes. If he has 40 coins in all, how many nickels does he have?

a. $10x + 5x = 40$
b. $10x + 5x = 310$
c. $10x + 5(40 - x) = 310$
d. $10x + 5(310 - x) = 40$

_____ 3. Johnny bought 15 records for a total of \$67. If "rock" records cost \$5 each and "country" records cost \$4 each, how many of each type did he buy?

a. 5 rock and 10 country records
b. 7 rock and 8 country records
c. 8 rock and 7 country records
d. 9 rock and 6 country records

_____ 4. Two numbers differ by 17. $\frac{1}{4}$ of the larger number plus $\frac{2}{3}$ of the smaller number equals 18. What are the numbers?

a. 16 and 12
b. 24 and 15
c. 42 and 25
d. 32 and 15

Key to Self-Correcting Exercises in Lesson 14

Self-Correcting Exercise A

(a) $1.00
(b) $29.85
(c) $4.50
(d) $5.97
(e) $295.00
(f) $9.00
(g) $15.70

Self-Correcting Exercise B

1.

Items	Number	@	Total
$8 records	x	8	8x
$6 records	11 - x	6	6 (11 - x)

2.

Items	Number	@	Total
Hotdogs	2	2x	4x
Can of pop	1	x	x

3.

Items	Number	@	Total
$38 bats	x	38	38x
$49 bats	9 - x	49	49 (9 - x)

Self-Correcting Exercise C

(a) 50
(b) 10x
(c) 5k
(d) 10(k - 3) = 10K - 30
(e) 10b + 5(3b + 1) = 10b + 15b + 5 = 25b +5
(f) 25x + 5(x+2) = 25x + 10 = 35x + 10
(g) 10d + 25(d + 1) = 10d + 25d + 25 = 35d + 25

Self-Correcting Exercise D

(a)

Items	Number	@	Total
dimes	2x	10	10(2x)
quarters	x	25	25x

Equation: 10(2x) + 25x = 900

(b)

Items	Number	@	Total
dimes	2x + 10	10	10(2x + 10)
nickels	x	5	5x

Equation: 5x + 10(2x + 10) = 900

(c)

Items	Number	@	Total
dimes	x + 15	10	10(x + 15)
quarters	x	25	25x

Equation: 25x + 10(x + 15) = 885

(d)

Items	Number	@	Total
cents	2x + 36	1	2x + 36
nickels	x	5	5x

Equation: $(2x + 36) + 5x = 162$

Self-Correcting Exercise E

1. (a) $n + 1$
   (b) $n - 8$
   (c) $n + 9$

2. (a) $(2n + 1) + 3 = 2n + 4$
   (b) $(2n + 1) - 5 = 2n - 4$
   (c) $(2n + 1) + 1 = 2n + 2$

3. (a) $2n$
   (b) $n + 6$
   (c) $4n + 2$

Self-Correcting Exercise F

(a) $63 - x + 4(x + 15) = 234.$

(b) $55 - x = 2(x + 5)$

(c) $3n - 3 = 4(n - 3)$

(d) $3(80 - x) + (x + 30) = 242$

End of Lesson 14

LESSON RECORD FORM

0902 Math 9
Revised 11/90

FOR STUDENT USE ONLY

Date Lesson Submitted

_____

Time Spent on Lesson

_____

(If label is missing or incorrect)

File Number

_____

Lesson Number _____

FOR A.D.L.C. USE ONLY

Assigned Teacher: _____

Lesson Grading: _____

Additional Grading E/R/P Code: _____

Mark: _____

Graded by: _____

Assignment Code: _____

Date Lesson Received:

_____

Lesson Recorded _____

Student's Questions and Comments

Apply Lesson Label Here

Name

Address

Postal Code

Please verify that preprinted label is for correct course and lesson.

Teacher's Comments:

_____
Correspondence Teacher

St. Serv. 14-91

ALBERTA DISTANCE LEARNING CENTRE

MAILING INSTRUCTIONS FOR CORRESPONDENCE LESSONS

1. BEFORE MAILING YOUR LESSONS, PLEASE SEE THAT:

(1) All pages are numbered and in order, and no paper clips or staples are used.
(2) All exercises are completed. If not, explain why.
(3) Your work has been re-read to ensure accuracy in spelling and lesson details.
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MARKETING

In this lesson you will learn about:

- profit and loss
- discounts and taxes
- commission
- simple interest and compound interest

KEEPING SKILLFUL

EXERCISE 1 1. Change the following percents to decimal form.

(a) 5% (b) $8\frac{1}{2}\%$

2. Use the form $\frac{a}{b} = \frac{r}{100}$ to solve the following. (See Lesson 9)

(a) What is 45% of 900?

(b) What percent of 80 is 25?

MARKETING OF GOODS

People involved in marketing goods are manufacturers, wholesalers, retailers and consumers. A manufacturer is the one who makes a certain product, a wholesaler buys from the manufacturer and sells to a retailer (who could be a small store owner), and the retailer then sells to a consumer (the person who buys a product). All of the above people are concerned about profit, interest, commission, tax and discount.

Profit is found by subtracting the expenses from the cost. It is really the amount you have earned after your expenses are subtracted.

Interest is money paid for using borrowed money. Banks lend money at a particular rate of interest. This rate of interest is what you have to pay for using the bank's money.

Commission is the amount of money earned for selling an item. It is based on a certain percentage. Car salesmen and real estate salesmen earn money by receiving a certain percentage of the selling price of the items sold.

Tax is also based on a certain percentage. It is an amount of money paid to the government. Examples of money paid to the government are income tax and sales tax.

Discount is an amount deducted from the price of an item. Retailers allow for discounts in order to promote more business.

PROFIT AND LOSS

To find profit or loss, the selling price and the cost price must be known. The selling price is what you have actually sold an item for. The cost price is what you originally paid to buy the item.

If the selling price is greater than the cost price, a profit is made. However, if the selling price is less than the cost price, no money is made and there is a loss. Nobody enjoys losing money, but sometimes it does happen.

Profit	=	Selling Price	-	Cost Price
Loss	=	Cost Price	-	Selling Price

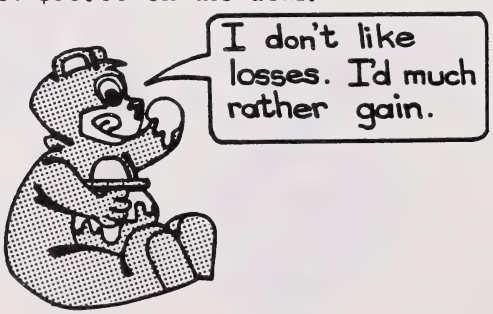
Example Tony sold a bicycle for $185.00. He had the bicycle for a year and originally paid $220.98 to purchase it. What was his profit or loss?

Cost price = $220.98 Selling price = $185.00

Since Tony paid more for the bicycle than he sold it for, he had a loss rather than a profit.

$$\text{Loss} = 220.98 - 185.00$$
$$= \$35.98$$

Tony lost $35.98 on the deal.



EXERCISE 2 Find either the amount of profit or loss.

1. Julius purchased four television sets at a cost of $825 each. Then, he sold them individually for $910, $775, $835 and $750. How much profit or loss did he have?

2. Ms. Bright paid $104.75 for 15 records. She decided to sell each record for $7.49. Will she have a profit or a loss? How much?

DISCOUNTS AND TAXES

Discounts are found by taking a certain rate (percentage) off the selling price.

$$\text{Discount} = \text{Rate} \times \text{Original Selling Price}$$

To find the new selling price the discount is now subtracted from the original selling price.

$$\text{New Price} = \text{Original Selling Price} - \text{Discount}$$

Example 1 A stereo was originally priced at \$425. If the store allows for a 15% discount, what is the stereo sold for?

$$\text{Discount} = \text{Rate} \times \text{Original Selling Price}$$

$$= 0.15 \times 425$$

Change the percent to decimal form 15% = 0.15

$$= \$63.75$$

$$\text{New Price} = \text{Original Selling Price} - \text{Discount}$$

$$= 425 - 63.75$$

$$= \$361.25$$

The selling price of the stereo is now \$361.25

Example 2 A colour television set was sold for \$650. If the original price of the television was \$799, what was the rate of discount allowed?

First, the amount of discount must be found before the rate can be determined.

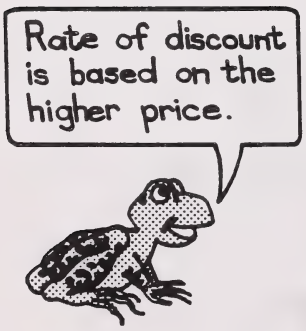
$$\text{New Price} = \text{Original Selling Price} - \text{Discount}$$

$$650 = 799 - \text{Discount}$$

$$650 + \text{Discount} = 799$$

$$\text{Discount} = 799 - 650$$

$$= \$149$$



$$\text{Discount} = \text{Rate} \times \text{Original Selling Price}$$

$$149 = \text{Rate} \times 799$$

$$149 \div 799 = \text{Rate}$$

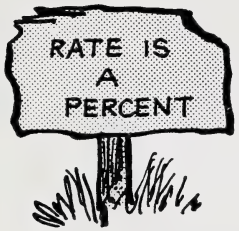
$$0.186 = \text{Rate}$$

To change a decimal to a percent, multiply by 100.

$$0.186 = 18.6\%$$

The rate of the discount is 18.6%.

Sales tax is very common throughout most of Canada. It is an amount that is added to the original selling price of a product.



$$\text{Sales Tax} = \text{Rate} \times \text{Selling Price}$$

$$\text{Cost of Product} = \text{Selling Price} + \text{Sales Tax}$$

However, income tax is a certain percentage taken off your earnings that is paid to the government.

$$\text{Income Tax Deducted} = \text{Rate} \times \text{Total Earnings}$$

Example 1 Sally wants to buy a new dress that costs \$99.99. If the sales tax is $4\frac{1}{2}\%$, how much will Sally pay for the dress?

$$\text{Sales Tax} = \text{Rate} \times \text{Selling Price}$$

$$\text{Sales Tax} = 4\tfrac{1}{2}\% \times 99.99$$

$4\frac{1}{2}\% = 0.045$

$$= 0.045 \times 99.99$$

$$= \$4.50$$

Rounded to the nearer cent.

$$\text{Cost of Product} = \text{Selling Price} + \text{Sales Tax}$$

$$\text{Cost of Dress} = 99.99 + 4.50$$

$$= \$104.49$$

Sally will have to pay \$104.49 for the new dress.

Example 2 Georgia received her pay cheque for the month of October. Her total earnings were $1500. She noticed that the income tax deducted was $450. What is her rate of deduction?

$$\text{Income Tax Deducted} = \text{Rate} \times \text{Total Earnings}$$

$$450 = \text{Rate} \times 1500$$

$$\frac{450}{1500} = \text{Rate}$$

$$0.30 = \text{Rate}$$

$$0.30 = 30\%$$

Multiply by 100 to change to a percent.

Georgia's rate of deduction is 30%.

SELF-CORRECTING EXERCISE A

Solve the following problems:

1. Dan bought a pair of jeans for $49.00. He paid an additional 7% sales tax. How much sales tax did he pay?

2. A $750 television is reduced $150. What is the rate of discount?

Check your answers with those on page 19 of this lesson. Correct any errors in your work.

EXERCISE 3 Do the following questions.

1. What would the total cost of a suit be if its selling price is $169.00 and the sales tax is 7%?

2. A discount of 15% is placed on all books that cost $15.00. How much would you have to pay for each book now?

3. The amount of income tax taken off Joanne's total earning for the year was $5500. If this represents a 25% deduction, what were her total earnings for the year?

COMMISSION

Manufacturers, wholesalers and retailers who sell their goods through sales representatives pay these people a certain percentage of the sales. This type of payment is called **commission**.

Commission = Rate × Sales

Example 1 Ambrose is a salesman for United Grain Grabbers. His sales for the year amount to $900 000. If he receives 5% commission on all sales, how much money will he earn?

$$\begin{aligned} \text{Commission} &= \text{Rate} \times \text{Sales} \\ &= 0.05 \times 900\ 000 \\ &= \$45\ 000 \end{aligned}$$

Ambrose would receive $45 000.

Example 2 Sam sold a used car and received $300.00 as a commission. If this represents a 6% commission, what did the car sell for?

$$\begin{aligned} \text{Commission} &= \text{Rate} \times \text{Sales} \\ 300 &= 0.06 \times \text{Sales} \\ 300 \div 0.06 &= \text{Sales} \\ \$5000 &= \text{Sales} \end{aligned}$$

The car sold for $5000.

SELF-CORRECTING EXERCISE B

Solve.

1. A car salesman receives $10\frac{1}{2}\%$ commission on his total sales. How much must he sell to earn $3150 commission ?

Check your answers with those on page 19 of this lesson. Correct any errors in your work.

EXERCISE 4 Do the following questions on commission. Write the general formula and show all of your work.

1. Wiliam works as a truck salesman on 5% commission for Less Cash Sales. Recently he sold a new pick-up truck for $13 545. How much did he earn?

EXERCISE 4 (continued)

2. A real estate agent wants to receive about $1800 each month. If he works on a 3% commission, what would his sales have to be for the month?

INTEREST

A knowledge of percents is necessary when calculating interest. You may want to review Lesson 7 before doing this section.

Interest is usually calculated on a percentage basis. In calculating interest you will use a special interest formula rather than the proportion, $\frac{a}{b} = \frac{r}{100}$, that you used in Lesson 9.

The special interest formula is sometimes called simple interest. The formula is $I = Prt$, where:

I is the interest.

P is the principal (that is, the amount borrowed or the amount deposited).

r is the rate, which is the percent expressed over 100 or, it may be in a decimal form.

t is the time for which the money is borrowed or deposited. (The units correspond to the length of time quoted in the rate.)

Simple interest is seldom used in financial matters today. However, it is the basis for the compound interest that is commonly used. It is necessary to be thoroughly familiar with simple interest.

Reference Study page 159 of *Holt Mathematics 3.*

Example 1 Bob Smithchuk borrowed \$7000 from his friend at a rate of 13% per year. Bob must pay out this loan within 3 years. How much interest is paid after 3 years?

$I = ?$; $P = 7000$; $r = 13\%$; $t = 3$ years

You are calculating the interest. *The amount borrowed.* *Change to decimal form.* *The time is expressed in years.*

$$I = P \times r \times t$$
$$= 7000 \times 0.13 \times 3$$
$$= \$2730$$

Bob will have to pay \$2730 interest on the \$7000.

This means that Bob's total payment to his friend after three years would be: $7000 + 2730 = \$9730$.

Example 2 Charlie Jones made a \$110 payment on his Chargex bill of \$432. He must pay $1\frac{1}{2}\%$ per month on the unpaid balance. How much interest will be due on Charlie's next bill?

$I = ?$; $P = 432 - 110$; $r = 1\frac{1}{2}\%$; $t = 1$ month

Unpaid balance. $= 322$ $= 0.015$

$$I = P \times r \times t$$
$$= 322 \times 0.015 \times 1$$
$$= \$4.83$$

Since the rate is "per month", the time is in months.

On next month's bill Charlie will be charged \$4.83 in interest.

SELF-CORRECTING EXERCISE C

Solve:

1. Jack invested \$500 at $8\frac{1}{4}\%$ simple interest. How many years will it take the balance to grow to \$912.50?

2. Jan wants to earn \$8000 interest in one year to pay for her trip to Europe. How much money would she have to invest at $12\frac{3}{4}\%$ simple interest?

Check your answers with those on page 20 of this lesson. Correct any errors in your work.

COMPOUND INTEREST

The topic of compound interest is really beyond the scope of this lesson. It is so much a part of everyday life that an introduction, at least, is desirable.

The difference between compound interest and simple interest is in the principal. In simple interest the principal is constant, whereas in compound interest the principal increases at each compounding.

For example, if you invest \$100 at 8% per annum for 10 years, the simple interest would be:

$$\begin{aligned} I &= P \times r \times t \\ &= 100 \times 0.08 \times 10 \\ &= \$80 \end{aligned}$$

However, if this money were invested at 8% compounded quarterly, your interest earnings would be \$120.80.

It is easy to see why lending institutions charge compound rather than simple interest for their loans.

Compounded quarterly means that the interest is calculated at the end of each quarter of a year (or every 3 months). Compounded semi-annually means that it is calculated every six months. Compounded annually would mean that the interest is found at the end of each year. For compound interest, the principal changes with each time period. At each compounding the interest is added to the principal to form the new principal for the next compounding.

Example \$1000 is invested at 12% for one year. Find the simple interest and the compound interest (if it is compounded quarterly).

Simple Interest

$$\begin{aligned} I &= P \times r \times t \\ &= 1000 \times 0.12 \times 1 \\ &= \$120 \end{aligned}$$

The simple interest earnings would be \$120 giving a total of \$1000 + \$120 = \$1120.

Example (continued)

Compound Interest

(a) After the first quarter:

$$I = 1000 \times 0.03 \times 1$$

One-quarter of 12% is 3%.

$$= \$30$$

At the end of the first quarter, the new principal will be \$1000 + \$30 = \$1030. This will be the principal for the second quarter.

(b) After the second quarter:

$$I = 1030 \times 0.03 \times 1$$

$$= \$30.90$$

At the end of the second quarter, the new principal will be \$1030 + \$30.90 = \$1060.90.

(c) After the third quarter:

$$I = 1060.90 \times 0.03 \times 1$$

$$= \$31.83$$

At the end of the third quarter, the new principal will be \$1060.90 + \$31.83 = \$1092.73.

(d) After one year:

$$I = 1092.73 \times 0.03 \times 1$$

$$= \$32.79$$

At the end of one complete year, you will have a balance of \$1092.73 + \$32.79 = \$1125.52. This is \$5.52 more than you would receive by simple interest.

This may seem like a small difference, but over a number of years the difference would be considerable. Also, the difference would be considerable if large sums of money were involved.

EXERCISE 5 Do the following question. Find the simple interest and then find the compound interest.

1. Don Jackson wants to invest $20 000 for two years. The MN Investment Company offers Don simple interest at 13% while the Western Credit Union offers Don 12% compounded semi-annually. Which is the better deal?

Simple Interest

(a) The MN Investment Company.

Compound Interest

(b) Western Credit Union.

(i) After the first six months:

I = ?; P = 20 000; r = 0.06; t = 1

Compounded semi-annually means that the rate of each period is 6%.

Total = I + P

= _____ + 20 000

= _____

(ii) After the first year:

I = ?; P = _____; r = 0.06; t = 1

Use the total from part (ii).

Total = I + P

= _____

EXERCISE 5 (continued)

(iii) After the first 18 months:

I = ?; P = _____; r = 0.06; t = 1

(Use the total from part (ii).

Total = I + P

= _____

(iv) After two years:

I = ?; P = _____; r = 0.06; t = 1

(Use the total from part (iii).

Total = I + P

= _____

(c) Which is the better deal?

_____

_____

KEY TO SELF-CORRECTING EXERCISES IN LESSON 15

SELF-CORRECTING EXERCISE A, Page 8.

1. Sales tax = rate × selling price
   = 0.07 × $49.00
   = $3.43

   He paid $3.43 sales tax.

2. Discount = rate × original selling price
   $150 = r × $750
   $\frac{150}{750} = r$
   0.2 = r

   The rate of discount is 20%.

SELF-CORRECTING EXERCISE B, Page 11.

1. Commission = rate × sales
   3150 = 0.105 × s
   $\frac{3150}{0.105} = s$
   30 000 = s

   His total sales must be $30000.

SELF-CORRECTING EXERCISE C, page 14.

1. $I = 912.50 - 500 = 412.50$ $P = 500$ $r = 8\frac{1}{4}\% = 0.0825$ $t = ?$

$$\begin{aligned} I &= P \times r \times t \\ 412.50 &= 500 \times 0.0825 \times t \\ 412.50 &= 41.25\ t \\ \frac{412.50}{41.25} &= t \\ 10 &= t \end{aligned}$$

It will take 10 years for Jack's savings to reach $912.50.

2. $I = 8000$ $P = ?$ $r = 12\frac{3}{4}\% = 0.1275$ $t = 1$ year

$$\begin{aligned} I &= P \times r \times t \\ 8000 &= P \times 0.1275 \times 1 \\ \frac{8000}{0.1275} &= P \\ 662745.098 &= P \\ 62745.10 &= P \end{aligned}$$

Jan will have to invest $62745.10.

END OF LESSON 15

LESSON RECORD FORM

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PROBLEM TYPES

In this lesson you will learn about:

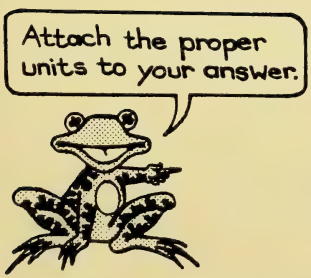
- averages
- direct variation
- variation in formulas
- problem solving by graphing

KEEPING SKILLFUL

EXERCISE 1 1. (a) Rearrange the formula $A = \frac{1}{2}bh$ to find h.

(b) Use your new formula from (a) to find the height of a triangle with base 6 cm and area 24 cm^2.

2. Use the formula $A = \frac{1}{2}h(a + b)$ to find A when h is 12 m, a is 7 m and b is 5 m.



In Lesson 13 you studied several formulas and used some of them to solve problems.

In this lesson there will be more emphasis on problems as well as a little more analysis of formulas.

AVERAGES

Average is a word that is often used in everyday conversations. Most people think of it as some sort of midpoint in a group of numbers or objects.

When mathematicians speak of average, they are referring to the arithmetic mean. This is found by dividing the sum of the objects to be averaged by the number of objects.

Example 1 In Grade 7 Lisa received the following final grades. What was the average of her marks?

Language Arts	75
Social Studies	70
Mathematics	85
Science	90
Health	75
Typewriting	60
Traffic Safety	95
Metrication	80
Ecology	55

$$\text{Average} = \frac{75 + 70 + 85 + 90 + 75 + 60 + 95 + 80 + 55}{9}$$

$$= \frac{685}{9}$$

There are 9 grades or final marks. So divide the total of all the marks by 9.

$$= 76 \text{ (to the nearer whole number)}$$

Lisa's average mark was 76.

Example 2 Find the average of the following numbers.

8, 9, -8, 7, -6, -12, -9, -5

$$\text{Average} = \frac{8 + 9 - 8 + 7 - 6 - 12 - 9 - 5}{8}$$

$$= \frac{24 - 40}{8}$$

$$= \frac{-16}{8}$$

$$= -2$$

8	- 9
9	- 6
7	-12
24	- 9
	- 5
	-40

The average of the numbers is -2.

EXERCISE 2 1. The daily high temperatures, recorded one week last February, in Beaverlodge are shown.

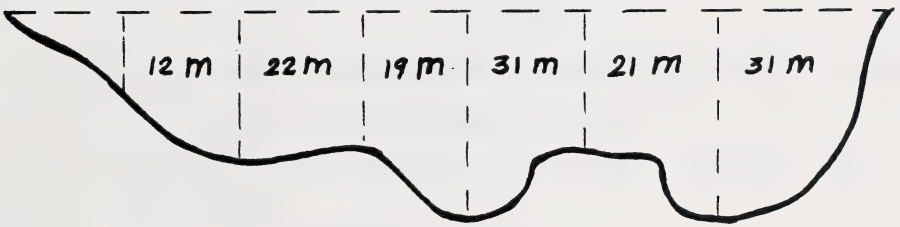
Monday	-4 °C	Thursday	-3 °C
Tuesday	-5 °C	Friday	0 °C
Wednesday	-5 °C	Saturday	2 °C

Sunday 4 °C

What was the average temperature for the week?

2. Ms. Wincton told her students that anyone whose average is 80 or better in mathematics will not have to write the final exam in that subject. Steve's grades in mathematics throughout the year were 83, 76, 82, 71, 90, 78, and 82. Will Steve have to write the final exam?

3. After measurements of the depth of Lake Wenewawa were taken, the following cross-section was drawn.



Find the average depth of the lake.

DIRECT VARIATION

In many everyday situations, two variable quantities are related in such a way that a change in one causes a change in the other.

For example, if the number of people attending a concert goes up, the amount of money taken in at the box office goes up and the number of empty seats goes down.

> When two quantities vary directly, an increase in one quantity causes a certain increase in the other. Direct variation also occurs when a decrease in one quantity causes a certain decrease in the other.

To solve direct variation problems, set up a proportion.

Example 1 The longer a car travels at a constant speed, the farther it goes. If such a car goes 315 km in 3 hours, how far will it have gone in 5 hours?

3 hours $\longrightarrow$ 315 km

5 hours $\longrightarrow$ x km

$$\frac{3}{5} = \frac{315}{x}$$

$$3x = 5 \times 315$$

$$x = \frac{5 \times \overset{105}{\cancel{315}}}{\cancel{3}_{1}}$$

$$x = 525$$

The car will travel 525 km in 5 hours.

Example 2 The Altagon Building casts a shadow of 9 m at the same time that Janice casts a shadow of 0.5 m. If Janice is 1.5 m tall, how tall is the building?

$$1.5 \text{ m tall} \longrightarrow 0.5 \text{ m shadow}$$

$$x \text{ m tall} \longrightarrow 9 \text{ m shadow}$$

$$\frac{1.5}{x} = \frac{0.5}{9}$$

$$0.5x = 9 \times 1.5$$

$$x = \frac{9 \times 1.5}{0.5}$$

$$x = \frac{13.5}{0.5}$$

$$x = 27.0$$

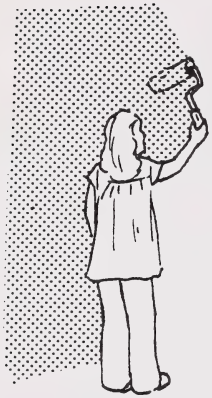
The Altagon Building is 27 m tall.

EXERCISE 3 Solve the following problems. Be sure to write a complete solution.

1. A car runs 100 km on 9 L of gas. How far will it run on 50 L?

EXERCISE 3 (continued)

2. It takes Karen 5 hours to paint 96 m^2 of wall area. How long will it take her to paint 700 m^2 of wall area? *(Round your answer to the nearer tenth of an hour.)*



3. If 2 large cans of succotash cost \$1.39, how much will 7 cans cost? *(Round your answer to the nearer cent.)*

VARIATION IN FORMULAS

You have just studied how one quantity can vary directly with another quantity. Now you will study how much a change in one part of a formula can affect a final answer.

Example (a) Find the area of a rectangle whose length is 30 m and whose width is 25 m.

$$\text{Area} = \ell \times w$$
$$= 30 \times 25$$
$$= 750 \text{ m}^2$$

(b) Find the area of a rectangle with the same width as the one in part (a), but twice the length.

$\ell = 2 \times 30 = 60$ m $w = 25$ m

$$\text{Area} = \ell \times w$$
$$= 60 \times 25$$
$$= 1500 \text{ m}^2$$

The area in part (b) is twice as much as the area of the rectangle in part (a).

EXERCISE 4 1. (a) Use the formula $P = 4s$ to find the perimeter of a square whose sides are each 25 cm long.

(b) Find the perimeter of a square whose sides are three times as long as those of the square in part (a).

(c) What is the relationship between your answers to (a) and (b)?

_____

_____

2. (a) Use the formula $A = \pi r^2$ to find the area of a circle whose radius is 7 km. ($\pi = \frac{22}{7}$)

(b) Find the area of a circle whose radius is three times as large as the radius of the circle in part (a).

(c) Is your answer to (b) three times your answer to (a)? _____

If not, what is the relationship?

_____

_____

EXERCISE 4 (continued)

3. (a) Find the area of a rectangle whose length is 60 m and whose width is 50 m.

(b) How does the length of this rectangle compare with the length of the rectangle in Example (a) on page 9?

_____

_____

(c) How does the width of this rectangle compare to the width of the rectangle in Example (a)?

_____

_____

(d) How does the area compare to the area found for the rectangle in Example (a)?

_____

_____

SOLVING PROBLEMS BY USING GRAPHS

In lesson 12 you learned to plot ordered pairs and linear equations on a Cartesian plane or graph. This same method may be used to see more clearly and readily the various possible solutions to a problem.

Example:

Jane owes Michael $1.00. He wants to be repaid using nickels and dimes only. What would be the largest number of coins Jane could use to repay the dollar? What would be the smallest number of coins Jane could use?

Solution: Let x and y represent the numbers of dimes and nickels involved respectively.

Then 10x and 5y represents the values of the two coins in cents.

The equation representing the relationship between the two coins is $10x + 5y = 100$.

In order to graph this relationship you should first solve for y.

$$10x + 5y = 100$$
$$10x + 5y - 10x = 100 - 10x$$
$$5y = 100 - 10x$$
$$\frac{5y}{5} = \frac{100 - 10x}{5}$$
$$y = \frac{100 - 10x}{5}$$

Then you must make a table of values. For example:

If $x = 1$, $y = \frac{100 - 10(1)}{5} = \frac{100 - 10}{5} = \frac{90}{5} = 18$

If $x = 2$, $y = \frac{100 - 10\ (2)}{5} = \frac{100 - 20}{5} = \frac{80}{5} = 16$

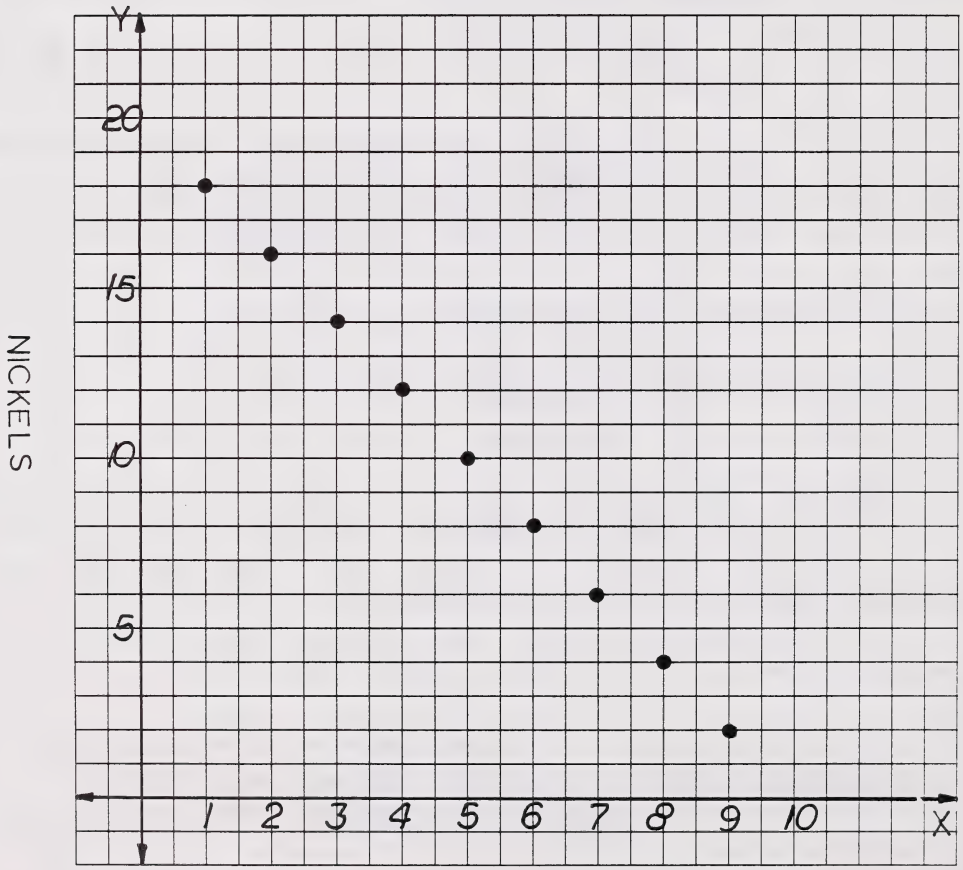
If $x = 3$, $y = \frac{100 - 10\ (3)}{5} = \frac{100 - 30}{5} = \frac{70}{5} = 14$

If $x = 4$, $y = \frac{100 - 10\ (4)}{5} = \frac{100 - 40}{5} = \frac{60}{5} = 12$

etc.

Thus the table of values and the graph illustrating this relationship will look like this.

x	1	2	3	4	5	6	7	8	9
y	18	16	14	12	10	8	6	4	2



The above illustrates all possible combinations of nickels and dimes. Now you can see that the smallest total number of coins that must be handled is 11 coins (9 dimes and 2 nickels). The greatest number, seen at the other end of the line, is 19 coins (1 dime and 18 nickels).

Exercise 5

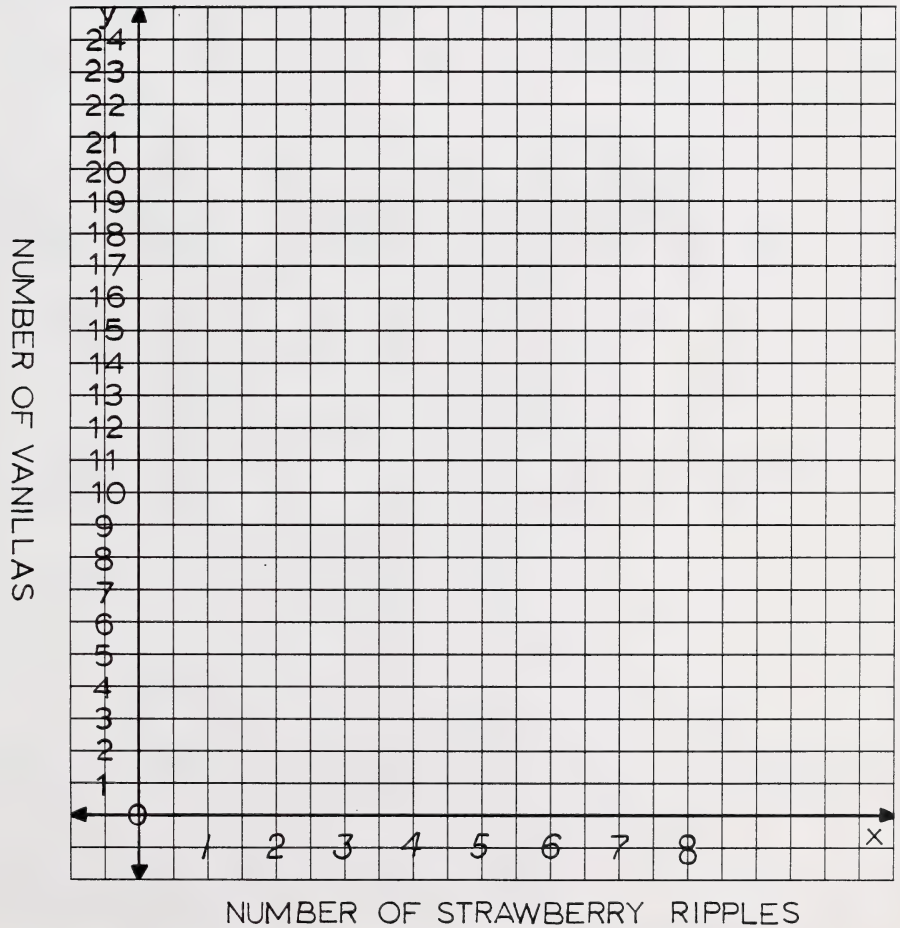
Mr. Billey has $48.00 to spend on ice cream for a party. The strawberry ripples cost $6.00 each and the vanillas $2.00 each on sale. Letting x equal the number of strawberry ripples and y the number of vanillas, represent this relationship with an equation. Graph the solution set of ordered pairs for this equation.

Equation:

Table of values:

x	0	1	2	3	4	5	6	7	8
y									

Graph:



Using your graph, answer the following questions.

1. What is the greatest number of strawberry ripples that could be bought? _____

2. What is the greatest number of vanillas that could be bought? _____

Intersections Of Two Linear Equations

Two linear functions drawn on the same graph may intersect and the point of intersection may solve a problem.

Example:

Darryl and Tom run a 50 metre race. Tom gets a 10 metre start because he is 2 metres per second slower than Darryl, who strides along at 7 metres per second. Who will win the race?

Solution:

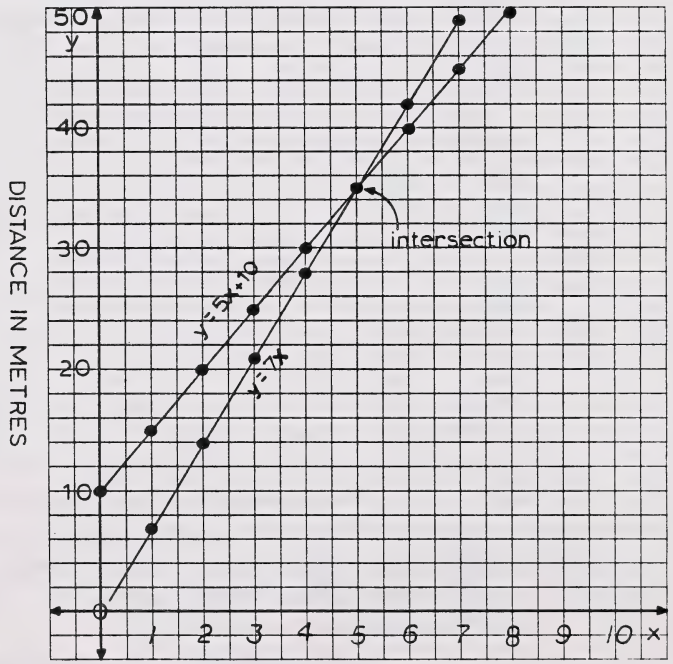
Let x seconds represent the time and y metres the distance. Because distance = rate x time, the equations to represent Darryl's progress is shown by y = 7x. Tom's speed is 5 metres per second but he has a 10 metre start so this equation is y = 5x + 10

y = 7x

x	0	1	2	3	4	5	6	7	8	9	10
y	0	7	14	21	28	35	42	49	56	63	70

y = 5x + 10

x	0	1	2	3	4	5	6	7	8	9	10
y	10	15	20	25	30	35	40	45	50	55	60



Darryl will win the race.

The intersection of the two graphs also clearly show that Darryl overtakes Tom in _____ seconds, _____ metres into the race.

Exercise 6

1. You may rent a car using either plan A or plan B. Plan A offers a flat rate of $5.00 per hour. Plan B calls for $12.00 down payment and $3.00 per hour. Graph the relationship between these two methods of renting.

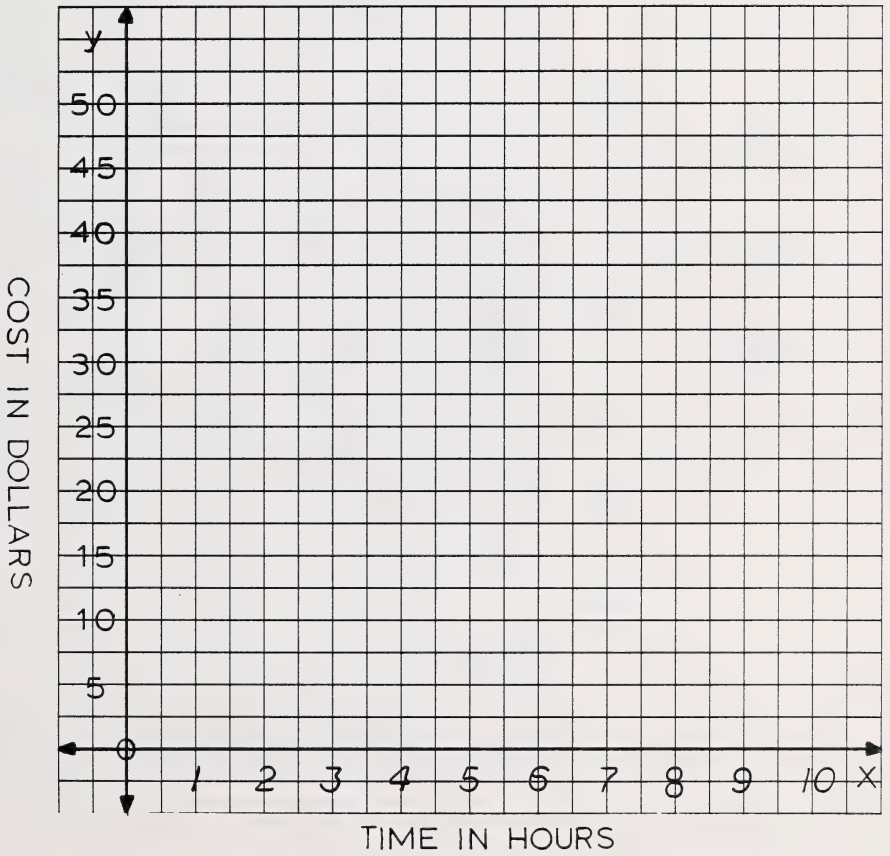
Let x represent the number of hours rented and y the cost of such rental.

Equation for Plan A: y = _____

Equation for Plan B: y = _____

x	1	2	3	4	5	6	7	8	9	10
y										

x	1	2	3	4	5	6	7	8	9	10
y										



a) Up to what point on the graph is Plan A more economical than Plan B? _____

b) At what point on the graph are Plan A and Plan B equally expensive. _____

2. Gloria leaves school riding her bicycle at a speed of 300 metres per minute. After she has gone 900 metres, Alex starts riding his bicycle at 500 metres per minute, trying to catch up to her. Graph these two relationships.

   Let x be the time for Alex (in minutes) and y be the distance in metres.

Equation for Gloria

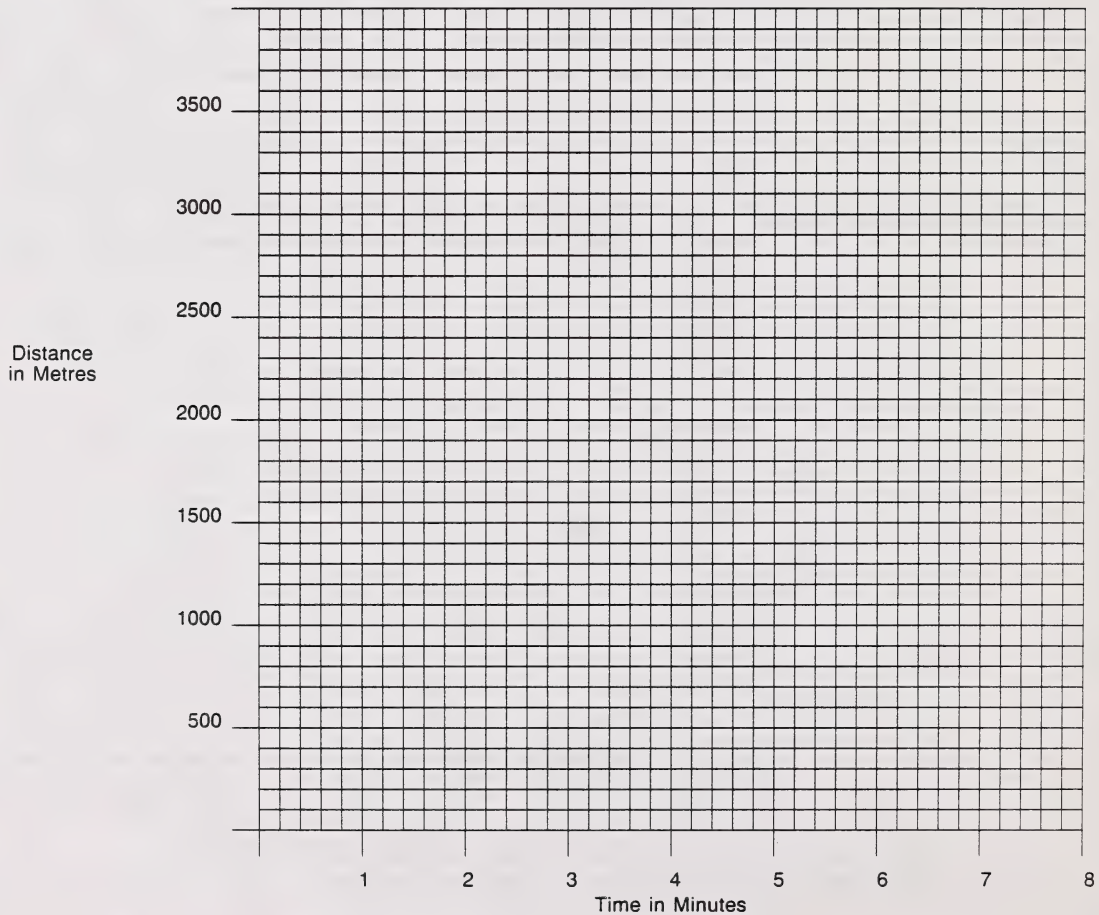
_____

x	0	1	2	3	4	5	6	7
y								

Equation for Alex

_____

x	0	1	2	3	4	5	6	7
y								



From this graph, we see that Alex will overtake Gloria after he has ridden _____ minutes and _____ metres.

END OF LESSON 16

LESSON RECORD FORM

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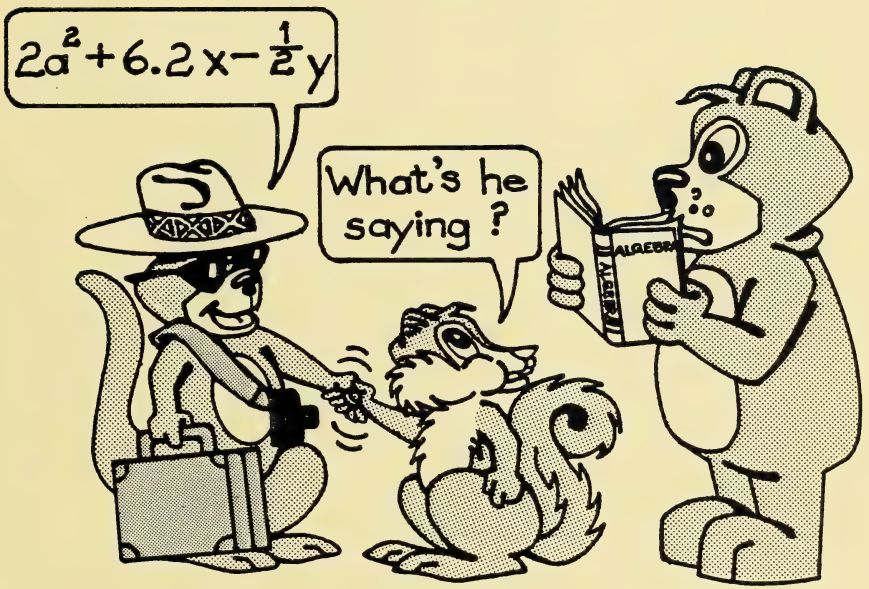
THE LANGUAGE OF ALGEBRA



In this lesson you will learn about:

- variables and constants
- equations and algebraic expressions
- terms and factors
- numerical and literal coefficients
- polynomials, nomomials, binomials, and trinomials
- degree
- descending order

KEEPING SKILLFUL

EXERCISE 1 Solve each of the following equations. Check your solutions.

1. $3x - 7 = -1$ Check:

2. $\frac{3}{4}y = 12$ Check:

3. $10y - 8 = 3y - 1$ Check:

ALGEBRAIC EXPRESSIONS

Algebra is the universal language of mathematicians and scientists. It deals with numbers, as arithmetic does, but it also uses letter symbols such as x, y, n, to represent numbers.

Variables

These symbols are called **variables**, because the value assigned to them will change or vary from one situation to another.

For instance, in one problem x may represent the number of tickets sold at a game; in the next problem, it could represent the number of kilograms of meat necessary for a dinner party, and so on.

Constants

The numbers are called **constants** because they always have the same value.

You have already dealt with algebra in your study of equations. For example, $4x + 7 = 8$ involves both variables and constants.

Algebraic Expression

On each side of the equals sign (as in $4x + 7 = 8$) is an **algebraic expression.** This is a very general name for almost any combination of letters, numbers, and signs.

The distinguishing feature of an equation is its equals sign.

Equation

Thus, an **equation** is a statement that two algebraic expressions are equal.

Exponents, while they are numbers, are not usually considered as constants.

EXERCISE 2 List the constants and variables for each of the following algebraic expressions.

Algebraic Expression	Constants	Variables
$-3x^2 + 7y + 6$	$-3, 7, 6$	x, y
$2x^2 - 5yz - 8$		
$7v^2 - 8v + 1$		
$35ab + 16c + 100$		

Factors Numbers which are multiplied together to form a product are called factors. Since variables represent numbers, they can be called factors as well.

Example $-3xy^2z = -3 \times x \times y \times y \times z$

factors

EXERCISE 3 List all the factors in the following products.

$-2a^2b^2$ means _____

$7x^4y^2$ means _____

$3(x - z)^2$ means $3 \times (x-z) \times (x-z)$

$-4xy^2$ means _____

$5(x - a)^3$ means _____

Terms

Many algebraic expressions are sums. The parts that are added together to form the sum are called **terms**.

Consider the algebraic expression $3x^2 + (-6x) + 4$. You will recall that another way to write this algebraic expression is $3x^2 - 6x + 4$.

It is understood that you are adding $3x^2$ and $-6x$ and 4. Thus, the terms of $3x^2 - 6x + 4$ are $3x^2$, $-6x$, and $+4$. Each term includes the sign ahead of it.

EXERCISE 4 List the terms in each of the following expressions.

Expression	Terms
$2x^2 - 3xy + 6yz$	$2x^2$, $-3xy$, $6yz$
$5a - b$	
$-9xy^2 + 6x$	
$\frac{1}{2}pq + 7$	

Distinction Between Terms and Factors

Be careful to use the words **term** and **factor** correctly.

1. Use the word **term** only when talking about things that are added or subtracted.

2. Use the word **factor** only when talking about things that are multiplied.

An algebraic term is always written as the product of a number and usually one or more variables.

$-3x^2 = -3 \times x \times x$

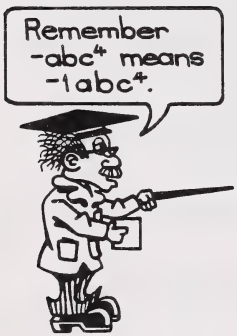
$y^3z = 1 \times y \times y \times y \times z$ ⟵ *y^3z means, $1y^3z$.*

Numerical and Literal Coefficients

The **numerical coefficient** of a term is the number factor of that term. It includes the + or - sign.

The **literal coefficient** of a term contains all the variable factors of the term. This includes the exponents on the variables.

Examples



Term	Numerical Coefficient	Literal Coefficient
$-\frac{3}{4}a^2b^2$	$-\frac{3}{4}$	a^2b^2
$-abc^4$	-1	abc^4
4	4	none

SELF-CORRECTING EXERCISE A

State the numerical coefficient and literal coefficient of each of the following terms.

Term	Numerical Coefficient	Literal Coefficient
$-3m^2n$		
x^3		
$\frac{-xy^2}{5}$		

Check your answers with those on page 20 of this lesson. Correct any errors in your work.

EXERCISE 5 For each of the following expressions state:

(a) the number of terms,

(b) the numerical coefficient of each term,

(c) the literal coefficient of each term.

1. $2a^2 - 4b^2 + c^2$ Number of terms? 3

Term	Numerical Coefficient	Literal Coefficient
$2a^2$	2	a^2
$-4b^2$	-4	b^2
c^2	1	c^2

2. $5x^3 - 4z^3$ Number of terms? _____

Term	Numerical Coefficient	Literal Coefficient

3. $13xyz$ Number of terms? _____

Term	Numerical Coefficient	Literal Coefficient

Degree

The **degree** of a term is the sum of the exponents of its variables.

Examples

$2x^2y^2$ has degree 4.

$(2 + 2 = 4)$

$4z^3$ had degree 3.

$-abc^4$ has degree 6.

$(1 + 1 + 4 = 6)$ *Remember:* $a = a^1$

Constant Term

A term of degree zero is called a **constant term.**

4 is said to be of degree zero because it can be written as:

$$4x^0 \quad \text{or} \quad 4y^0 \quad \text{or} \quad 4b^0$$

since x^0 or y^0 or $b^0 = 1$

Thus, all numbers by themselves are constant terms.

EXERCISE 6 State the degree of each of the following terms.

1. $2m^2$ _____
2. $15r^3t$ _____
3. $3x^5y^2$ _____
4. $\frac{1}{4}xy$ _____
5. $75x^2y^2z$ _____
6. $6y^{17}$ _____
7. 8 _____

POLYNOMIALS

A polynomial is a special kind of algebraic expression. Most of the expressions used in this lesson are polynomials.

In fact, a great deal of the algebra you will learn is concerned with the study of these forms of algebraic expressions.

A polynomial is an expression consisting of a single term or the sum of several terms. Each term of a polynomial must satisfy <u>one</u> of the following conditions:

		Example
1.	It is a number.	36
2.	It is a variable whose exponent is a whole number.	m^{23}
3.	It is the product of variables whose exponents are whole numbers.	x^2y^4z
4.	It is the product of a number and one or more variables whose exponents are whole numbers.	$36x^4$ $2y^4m^7$

Note that while the exponents of the variables in a polynomial must be whole numbers, there is no restriction on the numerical coefficient.

These are polynomials.	These are not polynomials	Why not?
$\sqrt{3x^2} + 1.68y$	$3\sqrt{x} + 2\sqrt{y}$	$\sqrt{x} = x^{\frac{1}{2}}, \sqrt{y} = y^{\frac{1}{2}}$ The exponents of the variables are not whole numbers.
$-4x^2$	$\frac{-4}{x}$	$\frac{-4}{x} = -4x^{-1}$ The exponent of the variable is not a whole number.

CLASSIFICATION OF POLYNOMIALS

A polynomial may be classified according to the number of terms it contains, the number of variables involved, or its term of highest degree.

Classifying According to Number of Terms

Examples

1. $x^2 - 5xy$ is a two term polynomial.

2. $a^3 + 3a^2 - 3a + 7$ is a four term polynomial.

Polynomials with one, two, or three different terms are given special names. No special names are assigned to polynomials with more than three terms.

Monomial

A **monomial** is a polynomial with one term.

Examples

The following expressions are monomials. Note that each expression consists of exactly one term.

1. -8
2. $4a^2b$
3. $-xyz$
4. $\frac{1}{2}a^3$

EXERCISE 7

Put a check mark beside each polynomial below that <u>is a monomial</u>.

1. $32z^3$	✓	5. $5xy + z$	_____
2. $a^2 - ab$	_____	6. $5xyz$	_____
3. a^2b	_____	7. $-a^2b^3c^5$	_____
4. $-8abc$	_____	8. $b^3 - a^2b^5$	_____

Binomial

A **binomial** is a polynomial with two terms.

Examples

The following expressions are binomials. Note that each expression consists of exactly two terms.

1. $-8x + 7y$ *Consists of the terms $-8x$ and $7y$.*

2. $a^2 - b^2$ *Consists of the 2 terms a^2 and $-b^2$.*

3. $-6x^4y^3 - 1$ *Consists of the 2 terms $-6x^4y^3$ and -1.*

EXERCISE 8 Put a check mark beside each polynomial below that is a binomial.

1. $5a^2 - 2a$ _____
2. $-6a^2bc$ _____
3. $x - y$ _____
4. $2x - 6y$ _____
5. $z^3 + z - 1$ _____
6. $a^3b - c$ _____
7. $x^2 + x$ _____
8. $-6 + 3m$ _____

Trinomial

A **trinomial** is a polynomial with three terms.

Examples

The following expressions are trinomials. Note that each expression consists of exactly three terms.

1. $-x + 5y - 10$ *Consists of the 3 terms $-x$, $5y$, and -10.*

2. $z^2 - 5yz + y^2$ *Consists of the 3 terms z^2, $-5yz$, and y^2.*

3. $2a - b - c$ *Consists of the 3 terms $2a$, $-b$, and $-c$.*

EXERCISE 9 Put a check mark beside each polynomial below that is a trinomial.

1. $a^2 - 5a$ _____
2. $a^2 + a - 5$ _____
3. $3xy$ _____
4. $x^2 - 5xy + y^2$ _____
5. $6z^2 + 3z - 1$ _____
6. $a + b - 2c + 1$ _____
7. $a^2 + b^2 + c^2$ _____
8. $a^2b^2c^2$ _____

9. Classify each of the following polymonials as being a "monomial", "binomial", or "trinomial".

	Classification
(a) $b^2 - 5b$	
(b) $2m^2 - 3mn + 4n^2$	trinomial
(c) $7a^2b^3c^4$	
(d) $y^3 - 3y^2 - 7$	
(e) $6y^2 + 2y - 8$	
(f) $z - 3x$	
(g) $-3xz$	
(h) $b^2c^2 - 2b^2 + 4c^2$	
(i) $b^3c - 2b^2$	

Polynomials of more than three terms have no special name. They are called simply polynomials ("poly" means an indefinite number).

Classifying According to Number of Variables

Examples The following expressions are polynomials in one variable.

1. $-m + 2$ *Involves the one variable, m.*

2. $y^2 + 3y - 7$ *Involves the one variable, y.*

3. $3r^3 - 5r^2 + 7r$ *Involves the one variable, r.*

4. $x^4 + x^3 - x^2 + x$ *Involves the one variable, x.*

The following expressions are polynomials in two variables.

1. $a^2 - 2ab + b^2$ *Involves the two variables, a and b.*

2. $y^3 + z^3$ *Involves the two variables, y and z.*

3. mn^4 *Involves the two variables, m and n.*

4. $x^4 + 4x^3y + 6x^2y^2 + xy^3 - y^4$ *Two variables, x and y.*

The following expressions all involve more than two variables.

1. $x^3 + y^3 + z^3$ *3 variables*

2. $3a^2bc^2d^3$ *4 variables*

3. $3rs + 2rt + t - 3w$ *4 variables*

4. $2p + 3q + 5r + 2s - 5t$ *5 variables*

EXERCISE 10 1. State the number of variables involved in each polynomial.

	Number of Variables
(a) $6x^2 - 5xy + 7y^2$	
(b) $-4xy^2 + 2z$	
(c) $a^3b + 2a^2b^2 - 7ab + b^4$	
(d) $x^2 - 2xz + y^2$	
(e) $a^2 + 2ab + bc + 2cd - d^2$	
(f) $a^3 - 3a^2 + 2a + 9$	

2. Classify each polynomial according to the number of terms it contains and the number of variables involved.

Polynomial	Classification (By Number of Terms)	Classification (By Number of Variables)
(a) $-x^2 + 2x$	binomial	1
(b) $6y^2 - 7yz + x^2$		
(c) $a^2 - 2bc + c^2 - b^3$		
(d) $m^4 - 3m^3 + 2m^2 - m$		
(e) $32ab^2c$		
(f) $a^2b + cd^2$		

Classifying According to Degree

In order to find the degree of a polynomial, you must find the degree of each term. The term with the highest degree determines the degree of the polynomial.

Read page 8 if you have forgotten how to find the degree of a term.

Example 1 What is the degree of $a - 5ab + b$?

a is of degree 1.

$-5ab$ is of degree 2.

$+b$ is of degree 1.

Since the second term has the highest degree, it determines the degree of the trinomial. Therefore, $a - 5ab + b$ is a trinomial of degree 2.

Example 2 What is the degree of $x^2y + 4xy^2 - 5x^2y^3 - y^3$?

x^2y is of degree 3.

$+4xy^2$ is of degree 3.

$-5x^2y^3$ is of degree 5.

$-y^3$ is of degree 3.

Since the third term has the highest degree, it determines the degree of the polynomial. Thus, $x^2y + 4xy^2 - 5x^2y^3 - y^3$ is a polynomial of degree 5.

EXERCISE 11 1. Classify each polynomial according to degree by filling in the blanks in the chart.

Polynomial	Degree of Terms (in order, from left to right)	Degree of Polynomial
$3x^2 - 5x + 9$	2, 1, 0	2
$6x^2y - 3y^2$		
$3mn - 5m + 6$		
$x^3 + 5x^2 - 2x + 8$		
$-3a + 7b - 6c$		
$x^2 - y^3 + 1$		
$3ab + b^2 - 2b + a^2$		

2. Give an example of each of the following polynomials.

a. A third degree binomial in two variables.

$4x^3 - 6y^2$

b. A fourth degree monomial in one variable.

_____

c. A first degree trinomial in three variables.

_____

d. A constant term. _____

3. Fill in the blanks in the following chart. For each polynomial, you must classify it according to the number of terms, number of variables, and its degree.

Polynomial	By Number of Terms	By Number of Variables	Degree
$-8x^2y + 2y^2 + 3$	*trinomial*	2	3
$-x^2 + 2yz$			
$m^3 - 3m^2n$			
$4abc$			
$2x + 9$			
$y^2 + 3y + 7$			
-12			
$2xy + 3yz$			
$2ab + 3cd^2 + 8$			
$2a - 3b + 5c$			
$-3x^2$			
$y^3 - 3y^2 + 4$			

DESCENDING ORDER

For some purposes such as division it is advisable to arrange a polynomial in a particular order. The most common order used is **descending order of degree.**

Example Arrange the following polynomial in descending order of degree.

$$12 - x^3 - 29x^2 + 14x^4$$

$$0 \quad 3 \quad 2 \quad 4$$

Degree of each term.

Rearranged in descending order, the polynomial becomes:

$$14x^4 - x^3 - 29x^2 + 12$$

$$4 \quad 3 \quad 2 \quad 0$$

Degree of each term.

Remember that the sign ahead of the term goes with the term.

EXERCISE 12 Arrange each of the following polynomials in descending order of degree.

1. $2a^3 + 10 - 16a - 39a^2 + 15a^4$

_____

2. $x - 7x^2 + 5$

_____

3. $30 - 11a + a^2$

_____

KEY TO SELF-CORRECTING EXERCISES IN LESSON 17

SELF-CORRECTING EXERCISE A, page 6.

Term	Numerical Coefficient	Literal Coefficient
$-3m^2n$	-3	m^2n
x^3	1	x^3
$\frac{-xy^2}{5}$	$-\frac{1}{5}$	xy^2

END OF LESSON 17

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0902 Math 9
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Time Spent on Lesson

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(2) All exercises are completed. If not, explain why.
(3) Your work has been re-read to ensure accuracy in spelling and lesson details.
(4) The Lesson Record Form is filled out and the correct lesson label is attached.
(5) This mailing sheet is placed on the lesson.

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